# **CONTEMPORARY MATHEMATICS**

### 181

## The Čech Centennial

A Conference on Homotopy Theory June 22–26, 1993 Northeastern University

> Mila Cenkl Haynes Miller Editors



American Mathematical Society

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# CONTEMPORARY MATHEMATICS

181

### The Čech Centennial

A Conference on Homotopy Theory June 22–26, 1993 Northeastern University

> Mila Cenkl Haynes Miller Editors



American Mathematical Society Providence, Rhode Island

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### Eduard Čech

June 29, 1893–March 15, 1960



Photo courtesy of the Czech Academy of Sciences

#### Preface

The Čech Centennial Homotopy Theory Conference was organized to commemorate the one-hundredth anniversary of the birth of Eduard Čech. The conference was held at Northeastern University in Boston between June 22–26, 1993. The organizers were Mila Cenkl, Mike Hopkins, Sol Jekel and Haynes Miller.

The main topics of the conference were the most recent results in the stable and unstable homotopy theory. The conference was attended by over one hundred mathematicians from around the world. In addition to the eleven plenary lectures there were thirty-three papers presented in two parallel sections.

This volume contains papers which were either presented or solicited at the conference. All papers were refereed, and we are glad to have this opportunity to thank the many referees who helped select and improve these papers.

We owe many thanks to Ms. Donna Marlowe and to all other staff members of the Department of Mathematics at Northeastern for making the conference and local arrangements a memorable success.

We acknowledge, with thanks, the generous support from Northeastern University, MIT, National Science Foundation, and the American Czech-and-Slovak Education Fund.

Finally we are grateful to Ms. Donna Harmon from the office of the American Mathematical Society for her fine secretarial support in assembling the volume.

> Mila Cenkl Haynes Miller

#### Eduard Čech

It was in the spring of 1952 when, as a high school student, I first had the opportunity to meet Eduard Čech in person. He gave the opening address to the participants of the first Czechoslovak Mathematical Olympiad, a competition Čech himself had established the previous year. At that time I had already been influenced by his approach to mathematics for several years, as many of the textbooks in use were written either directly by him or in collaboration with his colleagues.

Eduard Cech was a professor of mathematics at Charles University in Prague and also a member of the Czechoslovak Academy of Sciences. He was not only one of the greatest Czech mathematicians, whose research in topology and differential geometry had a lasting impact on the directions in those fields, but he was also a very influential teacher and mentor.

Eduard Čech was born on June 29, 1893 in Stračov in northeastern Bohemia, about 100 miles from Prague. He studied at Charles University and received his doctoral degree there in 1920. At that time his interest was mainly in the study of local invariants of submanifolds of a projective space. This work and his collaboration with Q. Fubini culminated in two books on projective differential geometry. In the late twenties, his broad interests in mathematics were already focused on problems in topology.

What struck me most profoundly about Čech when I was an undergraduate at Charles University was his approach to mathematics, mathematical thinking and his unique way of reading a book in mathematics. For example, after he read a theorem he would proceed to prove the statement himself before looking up the proof in the text.

Mathematics with all of its aspects was Čech's life. He had a remarkable capacity to focus on a chosen task. He had a great intuition, but was never satisfied with any result without an exact proof. He was a perfectionist. In 1926 he published a book on projective differential geometry (in Czech), where he attempted to present a rigorous treatment of differential geometry several years before the appropriate tools where invented.

Apart from his strict approach to mathematics, Čech would occasionally recount stories from his many experiences. Once he mentioned a book of Lefschetz he had read before his trip to Princeton for a year-long stay at the Institute in

#### EDUARD ČECH

1935. He said that after reading the book he wrote to Lefschetz that the book was great, that all the theorems were correct but that none of the proofs were quite right. He also sent his proofs to Lefschetz. Years later, in 1968, when I had the opportunity to ask Lefschetz about this, he recalled the incident, saying that, "Oh yes, I remember that. Čech was quite an extraordinary young man."

Čech's paper which most closely reflects the main topic of the Čech Centennial Homotopy Conference is the brief communication on higher homotopy groups presented at the International Congress of Mathematicians held in Zürich in 1932. In that paper Čech defined the higher homotopy groups. Commenting on Čech's definition, P. S. Alexandrov wrote in 1961: "This definition did not meet with the attention it merited; in fact, the commutativity of these groups for dimensions greater than one was criticized. We must express our admiration at the intuition and talent of Professor Čech, who defined the homotopy groups years before W. Hurewitz."

Cech published 30 papers in topology between 1930 and 1938. All those papers were reprinted in the 1968 text, Topological Papers of Eduard Čech, by Academic Publishing House of the Czechoslovak Academy of Sciences. The papers containing some of Čech's major contributions to topology are: On bicompact spaces, Ann. of Math. **38** (1937), 823–844; Sur la théorie de la dimension, C.R. Acad. Sci. Paris **193** (1931), 976–977; Sur la dimension des espaces parfaitement normaux, Bull. Internat. Acad. Tchèque Sci. **33** (1932), 38–55; Contribution to dimension theory (in Czech), Časopis Pěst. Mat. Fys. **62** (1933), 277–291; Théorie générale de l'homologie dans un espace quelconque, Fund. Math. **19** (1932), 149–183; Les groupes de Betti d'un complexe infini, Fund. Math. **25** (1935), 33–44; Multiplication on a complex, Ann. of Math. **37** (1936), 681–697; Höherdimensionale Homotopiegruppen, Verh. des int. Kongr. Zürich **2** (1932), 203. An extended biography and a bibliography of Čech's work can be found in the book The Mathematical Legacy of Eduard Čech, published by Academia, Praha, 1993.

Starting in 1939 all the universities in Bohemia and Moravia were closed for the duration of the German occupation. After the war, in 1945, Čech returned to problems in differential geometry. He played a major role in the reconstruction of mathematical life in Czechoslovakia. He was instrumental in founding the Mathematical Institute of the Czechoslovak Academy of Sciences in 1950 and the Mathematical Institute at Charles University in 1956. Čech remained very active in all aspects of mathematics in Czechoslovakia until his death on March 15, 1960.

Mila Cenkl





#### List of Participants

The numbers in the parantheses refer to the group picture

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A. Baier	P. Hirschhorn (12)	J. Neisendorfer (73)
M. Bendersky (67)	M. Hopkins	A. Nofech $(37)$
T. Bisson $(39)$	M. Hovey (38)	J. Palmieri (25)
D. Blanc $(9)$	T. Hunter $(42)$	C. Peterson (69)
M Boardman (18)	I Hunton (43)	D Ravenel (81)
B. Botvinnik (65)	M Intermont (48)	C. Rezk $(4)$
E Brown $(75)$	B Jardine (21)	D Robenson
B Bruner $(49)$	J Jones (76)	H. Sadofsky (11)
I  Bureš (77)	S. Jekel	$\begin{array}{c} \text{I. Shank} (29) \end{array}$
E Campbell (20)	B. Kane (28)	P Shick $(61)$
C. Casacuberta $(55)$	M Kapranov	D Shimamoto $(40)$
B Cenkl $(79)$	K Kelly (46)	B Shinley
D. Christeasen $(36)$	S. Kochman (35)	E Siegel
E. Cohen $(3)$	$\mathbf{L}  \mathbf{Kriz} \ (1)$	P Silberbush (74)
O Cornea $(50)$	P Landweber $(71)$	J Silverman (47)
D Davis	L Langsetmo (44)	W Singer (66)
E Devinatz (70)	C N Lee $(45)$	D Stapley $(6)$
I. Dolinak	K Lesh $(51)$	N. Strickland (10)
J. Dorrington	R. Levi (72)	A. Szenes
W. Dwver (83)	A. Liulevicius $(33)$	D. Sinha
A. Elmendorf (22)	B. Mao (19)	S. Stolz $(62)$
E. Farjoun (41)	H. Marcum (5)	A. Suciu
V. Franjou (7)	D. Massev	R. Thomson (15)
E. Getzler	J. McCleary (53)	J. Turner (82)
V. Giambalvo (56)	J. McClure (24)	A. Viruel (59)
P. Goerss $(17)$	J. P. Mever (54)	M. T. Wang
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-	• •	

#### Titles of $Talks^1$

*	F. Cohen	Combinatorial Group Theory in Homotopy.
*	S. Stolz	Manifolds of Positive Scalar Curvature.
	J. Palmieri	Nilpotence and Periodicity over Steenrod Algebra.
	K. Monks	Polynomial Modules over the Steenrod Algebra
		and Conjugation in the Milnor Basis.
	J. Silverman	On the Action of Steenrod Squares on
		Polynomial Algebras.
	J. Møller	Centers of p-compact Groups.
	V. Gorbunov	On Spectra $(CP \wedge MSp)_{-\infty}$ and $(P \wedge MSp)_{-\infty}$ .
	R. Levi	On Homotopy Uniqueness of Certain Loop Spaces.
*	I. Kriz	Algebraic Constructions of Spectra.
*	C. Wilkerson	Centers and Semisimplicity for p-compact Groups.
*	J. Bureš	Life and Work of Eduard Čech.
*	M.Kapranov	Koszul Duality for Operads.
*	R. Bott	Ruminations on Knots.
	A. Elmendorf	A Conceptual Approach to Rings and Module
		Spectra.
	P. Schick	A Corrected Version of the Telescope
		Conjecture.
	B. Mao	BP Analogue of Lin's Theorem.
	A. Nofech	Localization of Inverse Limits.
	D. Blanc	Recognition Principles of Mapping Spaces.
	J. Neisendorfer	On the Weak Rational Homotopy Type of
		Tiny Bubbles.
	P. Goerss	Homotopy Fixed Points for Galois Groups.
	D. Davis	$v_1$ -Localizations of Finite Torsion Spectra
		and Spherically Resolved Spaces.
*	J. Jones	Floer's Infinite Dimensional Morse Theory
		and Homotopy Theory.

<sup>&</sup>lt;sup>1</sup>The titles are in chronological order. \* indicates a plenary talk.

*	W.Dwyer	Homotopy Theories.
	N. Minami	The Kervaire Invariant One and the
		Double Transfer.
	A. Suciu	Complements of Hyperplane Arrangements.
	J. McClure	K-theory and Andre-Quillen Homology.
	H. Sadofsky	Tate Homology and $v_n$ -periodicity.
	C. N. Lee	Homotopy Decomposition of the Classifying
		Space of Virtually Torsion Free Groups and
		Application.
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	J. Turner	Dyer-Lashof Operations in Second Quadrant
		Spectral Sequences.
	O. Cornea	Critical Point Theory and Cone-
		Decompositions.
	C. Casacuberta	Anderson Localization from a Modern
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	K. Xu	Spaces with Algebraically Closed Fields.
*	D. Ravenel	Telescopes, Past, Present and Future.
*	E. Farjoun	Cellular Inequalities and the Symmetric
		Product of Spaces.
	M.Hovey	A BP Analogue of Hopkins' Zeta
		Conjecture.
	M.Boardman	The Cohomology of a Point
		(and Some Other Spaces).
	E. Getzler	Operads and Moduli Spaces of Riemann Surfaces.
	$\mathbf{M}. \mathbf{W}$ instead	Another Look at Adam's Definition of the
		T-function.
	C. Peterson	Geometric Cohomology of Steenrod Algebra.
	B. Gray	Costable Homotopy.
	H. Marcum	Partial Whitehead Products.
	E. Campbell	Upper Triangular Invariants as a
		Module over the Dickson Invariants.
	D. Stanley	An Implication of Moore's Conjecture.

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#### **The Čech Centennial** Mila Cenkl and Haynes Miller, Editors

This book contains the refereed proceedings of the Čech Centennial Homotopy Theory Conference, organized to commemorate the 100th anniversary of the birth of Eduard Čech and held at Northeastern University in June 1993. Focusing on the theme of stable and unstable homotopy theory, the conference attracted more than one hundred mathematicians from around the world. With recent results by some of the leaders in the field, this book will provide readers with a look at important directions in current research in homotopy theory.

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