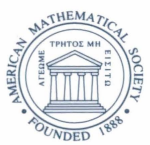
CONTEMPORARY MATHEMATICS

185

Multivariable Operator Theory

A Joint Summer Research Conference on
Multivariable Operator Theory
July 10–18, 1993
University of Washington, Seattle

Raúl E. Curto
Ronald G. Douglas
Joel D. Pincus
Norberto Salinas
Editors



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(Continued in the back of this publication)

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Contents

Preface

This volume consists of a collection of papers presented at the 1993 Summer Research Conference on Multivariable Operator Theory, held at the University of Washington in Seattle, under the auspices of the American Mathematical Society.

The articles represent contributions to a variety of areas and topics, which may be viewed as forming an emerging new subject, involving the study of geometric rather than topological invariants associated with the general theme of operator theory in several variables.

Beginning with J. L. Taylor's discovery in 1970 of the right notion of joint spectrum and analytic functional calculus for commuting families of Banach space operators, a number of significant developments have taken place. For instance, a Bochner-Martinelli formula has been generalized to commuting n-tuples in arbitrary C^*-algebras; various forms of the multivariable index theorem have been proved using non-commutative differential geometry and algebraic geometry; and systems of Toeplitz and Hankel operators on Reinhardt domains, bounded symmetric domains, and domains of finite type have been substantially understood from the spectral and algebraic viewpoints, including the discovery of concrete Toeplitz operators with irrational index.

These developments have been applied successfully to various types of quantizations, and functional spaces on Cartan domains and on pseudoconvex domains with smooth boundary have been thoroughly studied. A generalization of the Berger-Shaw formula to several variables has been proved; and connections with the local multiplicative Lefshetz numbers, analytic torsion, and curvature relations of canonically associated hermitian vector bundles have been established. Moreover, a sophisticated machinery of functional homological algebra suitable for the study of multivariable phenomena has been developed, and a rich theory for invariant pseudodifferential operators on domains with transverse symmetry has been produced.

Much of multivariable operator theory involves the interaction between the subspace geometry of defect spaces and algebraic K-theory. For one example, a multivariable index theorem corresponding to commuting pairs of elements with finite defect $A, B \in \mathrm{End}(H)$, where H is a vector space over an arbitrary field F, has emerged in connection with the Quillen algebraic K-theory. The

joint torsion $\tau(A, B; \mathcal{H})$ is a multiplicative Euler characteristic defined solely in terms of the homology of the Koszul complex $K_\star(A, B, H)$. It has been found to be the obstruction in the homotopy lifting problem from the classifying space $BGL(\mathrm{End}(H))^+ \to BGL(\mathrm{End}(H)/\mathcal{F}))^+$, where $\mathcal{F}$ is the finite rank ideal; for it has been shown that $\tau(A, B; \mathcal{H}) = \partial\{A + \mathcal{F}, B + \mathcal{F}\}$, where $\{A + \mathcal{F}, B + \mathcal{F}\}$ is the Steinberg element in $BGL(\mathrm{End}(H)/\mathcal{F}))^+$.

On a different direction, Calabi-like rigidity phenomena for analytically invariant subspaces of the Hardy and Bergman spaces have been discovered, and sheaf models for subnormal n-tuples have been formulated, which have led to a substantial understanding of their spectral properties. Results from polynomial convexity have been used to solve intriguing problems on joint quasitriangularity, and the polynomially hyponormal conjecture for single operators has been settled using ideas from joint hyponormality.

As probably expected during the early stages of a new subject, the recent years have seen the rise of many new approaches (all quite different) to multivariable operator theory, certainly connected, but with relationships not well understood. The subject has developed in several directions and with the aid of many and varied techniques, and a good number of the advances have been made through cross pollination among different areas of mathematics.

The principal goal of the conference was to provide a forum for the discussion of the actual connections among the various approaches, which one hopes will allow researchers to combine their efforts in finding an understanding of the above mentioned relationships and new directions for future research. These proceedings represent the products of those discussions.

The Editors
December 1994

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(*Continued from the front of this publication*)

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(See the AMS catalog for earlier titles)

Errata

The page numbers for the entries on the second page of the Table of Contents were inadvertently omitted. Here is the corrected page.

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