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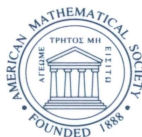
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Recent Developments in the Inverse Galois Problem

A Joint Summer Research Conference on
Recent Developments in the
Inverse Galois Problem
July 17–23, 1993
University of Washington, Seattle

Michael D. Fried, Coordinating Editor

Shreeram S. Abhyankar
Walter Feit
Yasutaka Ihara
Helmut Voelklein
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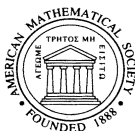
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Preface: Recent developments in the Inverse Galois Problem

MICHAEL D. FRIED

The conference **Recent Developments in the Inverse Galois Problem** occurred late July of summer 1993. Experienced **American Mathematics Society** staff was host to a turnout of over 75. Funding came from **AMS, NSF** and **Siam**. The organizing committee included Shreeram Abhyankar, Walter Feit, Michael Fried (chair), David Harbater, and Helmut Völklein. All four areas of the conference had intense mathematical activity from the period proceeding that summer. Some papers in this volume are the first publications of those events. As editor, I encouraged authors to emphasize connections to other conference areas. The central topic, of course, was the absolute Galois groups of the classical fields \mathbb{Q} , $\mathbb{Q}(T)$, $\mathbb{Q}_p(T)$ and $\bar{\mathbb{F}}_q(T)$. Here, \mathbb{Q}_p is the p -adic numbers; $\bar{\mathbb{F}}_p$ is the algebraic closure of the prime finite field \mathbb{F}_p . The authors of two papers here weren't at the conference. These fulfill aspects of Harbater's part of the proof of Abhyankar's conjecture.

The conference sought to recognize the maturity of several techniques. An enhanced review of Serre's book *Topics in Galois Theory* serves as a starting point for the conference. Practical applications in the moduli space approach through *braids* appear in the papers of Débes, Deschamps, and Völklein. Deschamps completes the paper of Débes on the following point. They start with a given finite group G , and apply a Fried-Völklein result to produce an infinite sequence of absolutely irreducible Hurwitz spaces over \mathbb{Q} with these properties. First: If even one had a \mathbb{Q} point, G would have regular realization as a Galois group over \mathbb{Q} . Second: *Each* has real points and \mathbb{Q}_p points for every prime p (an application of Harbater's patching method). Débes also answers questions posed by Dew's paper on local-global computations relations the field of definition to the *field of moduli* of covers. Wang's presentation of Mazur's conjecture influenced Débes. (Haran didn't contribute a paper. Still, his talk on joint work with Fried and Völklein emphasized the natural way Hurwitz spaces organize information on

real components of varieties. Thus, it pointed to the potential of Hurwitz spaces for Mazur's conjecture.)

Reverter and Vila continue old programs of Shih and Ribet collecting regular realizations of Galois groups from modular curves. Völklein produced various Chevalley group series of high rank over non-prime finite fields as Galois groups. He uses Tate modules to construct projective linear profinite groups as Galois groups. Matzat's parametric method and *GAR-realization* approach influenced him. Indeed, Matzat reviewed the continuing contributions of him and Malle to Shafarevich's conjecture. The classical (generalized) *rigidity hypothesis* for realizing a group G as the Galois group of a regular extension of \mathbb{Q} is that G has no center. Contributions of Crespo, and Swallow go deeply into examples that drop this assumption. Liedahl and Sonn bring new aspects of a mainstay application of the inverse Galois problem, *admissible groups*.

Fried, Ihara and Matsumoto cover totally new aspects of profinite use of braids. Ihara and Matsumoto bring the *Grothendieck-Teichmüller* group into the proceedings. They find a *canonical section* to the geometric-arithmetic fundamental group sequence attached to projective n -space minus its discriminant locus. Fried's paper ties to that approach with the introduction of *modular towers*, generalizations of sequences of modular curve covers. Thus, the inverse Galois problem collects under one umbrella modular curve applications and profinite aspects like pro-braids and *universal Frattini covers*. A separate paper by Matsumoto uses *Soulé elements* to find ranks of low quotients from a canonical pro- ℓ grading of $G_{\mathbb{Q}}$.

Abhyankar summarizes production of explicit quotients of the affine line fundamental group in positive characteristic. Elkies's gave a talk (without submitting a paper) updating classical elimination theory in characteristic p . This suggested the classical Noether approach to fixed fields of invariants of subgroups of S_n might work better in characteristic p than in characteristic 0. Data accrues that embarrassingly simple polynomials produce many sequences of Chevalley groups as such fundamental group quotients. Still, theoretical data clarifies you can't expect all quasi- p -groups to come from genus 0 covers. Guralnick and Neubauer give a characteristic 0 result in this direction. They (essentially) complete the classification of genus zero covers with affine geometric monodromy groups. Of immediate application is Müller's complete classification of geometric monodromy groups of polynomials $f \in \mathbb{C}[x]$ and of polynomials in $\mathbb{Q}[x]$.

Liu's well known rigid analytic geometry proof of Harbater's tactic for producing groups over valuation rings appears here. Harbater's older *Grothendieck patching method* gives a function field analog of Shafarevich's conjecture. Jarden answers an embedding problem question of Harbater distinguishing profinite Galois groups of uncountable fields. Galois descent and embedding problems are a natural part of the inverse Galois problem. Feit considered these in categorical generality. Seiler departs from Harbater's specialization (generalization) techniques; he uses deformations without totally degenerate specializations.

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Recent Developments in the Inverse Galois Problem
Shreeram S. Abhyankar, Walter Feit, Michael D. Fried,
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This book contains the refereed proceedings of the AMS-IMS-SIAM Joint Summer Research Conference on Recent Developments in the Inverse Galois Problem, held in July 1993 at the University of Washington, Seattle. A new review of Serre's *Topics in Galois Theory* serves as a starting point. The book describes the latest research on explicit presentation of the absolute Galois group of the rationals. Containing the first appearance of generalizations of modular curves, the book presents applications that demonstrate the full scope of the Inverse Galois Problem. In particular, the papers collected here show the ubiquity of the applications of the Inverse Galois Problem and its compelling significance. The book will serve as a guide to progress on the Inverse Galois Problem and as an aid in using this work in other areas of mathematics. This includes coding theory and other finite field applications. Group theory and a first course in algebraic curves are sufficient for understanding many papers in the volume. Graduate students will find this an excellent reference to current research, as it contains a list of problems appropriate for thesis material in arithmetic geometry, algebraic number theory, and group theory.

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