CONTEMPORARY MATHEMATICS

193

Moonshine, the Monster, and Related Topics

Joint Summer Research Conference on Moonshine, the Monster, and Related Topics June 18–23, 1994 Mount Holyoke College, South Hadley, Massachusetts

> Chongying Dong Geoffrey Mason Editors



American Mathematical Society

Other Titles in This Series

- 193 Chongying Dong and Geoffrey Mason, Editors, Moonshine, the Monster, and related topics, 1996
- 192 Tomek Bartoszyński and Marion Scheepers, Editors, Set theory, 1995
- 191 Tuong Ton-That, Kenneth I. Gross, Donald St. P. Richards, and Paul J. Sally, Jr., Editors, Representation theory and harmonic analysis, 1995
- 190 Mourad E. H. Ismail, M. Zuhair Nashed, Ahmed I. Zayed, and Ahmed F. Ghaleb, Editors, Mathematical analysis, wavelets, and signal processing, 1995
- S. A. M. Marcantognini, G. A. Mendoza, M. D. Morán, A. Octavio, and W. O. Urbina, Editors, Harmonic analysis and operator theory, 1995
- 188 Alejandro Adem, R. James Milgram, and Douglas C. Ravenel, Editors, Homotopy theory and its applications, 1995
- 187 G. W. Brumfiel and H. M. Hilden, SL(2) representations of finitely presented groups, 1995
- 186 Shreeram S. Abhyankar, Walter Feit, Michael D. Fried, Yasutaka Ihara, and Helmut Voelklein, Editors, Recent developments in the inverse Galois problem, 1995
- 185 Raúl E. Curto, Ronald G. Douglas, Joel D. Pincus, and Norberto Salinas, Editors, Multivariable operator theory, 1995
- 184 L. A. Bokut', A. I. Kostrikin, and S. S. Kutateladze, Editors, Second International Conference on Algebra, 1995
- 183 William C. Connett, Marc-Olivier Gebuhrer, and Alan L. Schwartz, Editors, Applications of hypergroups and related measure algebras, 1995
- 182 Selman Akbulut, Editor, Real algebraic geometry and topology, 1995
- 181 Mila Cenkl and Haynes Miller, Editors, The Čech Centennial, 1995
- 180 David E. Keyes and Jinchao Xu, Editors, Domain decomposition methods in scientific and engineering computing, 1994
- 179 Yoshiaki Maeda, Hideki Omoro, and Alan Weinstein, Editors, Symplectic geometry and quantization, 1994
- 178 Hélène Barcelo and Gil Kalai, Editors, Jerusalem Combinatorics '93, 1994
- Simon Gindikin, Roe Goodman, Frederick P. Greenleaf, and Paul J. Sally, Jr., Editors, Representation theory and analysis on homogeneous spaces, 1994
- 176 David Ballard, Foundational aspects of "non" standard mathematics, 1994
- 175 Paul J. Sally, Jr., Moshe Flato, James Lepowsky, Nicolai Reshetikhin, and Gregg J. Zuckerman, Editors, Mathematical aspects of conformal and topological field theories and quantum groups, 1994
- 174 Nancy Childress and John W. Jones, Editors, Arithmetic geometry, 1994
- 173 Robert Brooks, Carolyn Gordon, and Peter Perry, Editors, Geometry of the spectrum, 1994
- 172 Peter E. Kloeden and Kenneth J. Palmer, Editors, Chaotic numerics, 1994
- 171 Rüdiger Göbel, Paul Hill, and Wolfgang Liebert, Editors, Abelian group theory and related topics, 1994
- 170 John K. Beem and Krishan L. Duggal, Editors, Differential geometry and mathematical physics, 1994
- 169 William Abikoff, Joan S. Birman, and Kathryn Kuiken, Editors, The mathematical legacy of Wilhelm Magnus, 1994
- 168 Gary L. Mullen and Peter Jau-Shyong Shiue, Editors, Finite fields: Theory, applications, and algorithms, 1994
- 167 Robert S. Doran, Editor, C*-algebras: 1943–1993, 1994

(Continued in the back of this publication)

Moonshine, the Monster, and Related Topics

CONTEMPORARY MATHEMATICS

193

Moonshine, the Monster, and Related Topics

Joint Research Conference on Moonshine, the Monster, and Related Topics June 18–23, 1994 Mount Holyoke College, South Hadley, Massachusetts

> Chongying Dong Geoffrey Mason Editors



American Mathematical Society Providence, Rhode Island

Editorial Board

Craig Huneke, managing editor Clark Robinson J. T. Sta Linda Preiss Rothschild Peter M

J. T. Stafford Peter M. Winkler

The AMS-IMS-SIAM Joint Summer Research Conference in the Mathematical Sciences on Moonshine, the Monster, and Related Topics was held at Mount Holyoke College, South Hadley, Massachusetts, June 18–23, 1994, with support from the National Science Foundation, Grant DMS-9221892-001 and the National Security Agency, Grant MDA904-94-H-2045.

1991 Mathematics Subject Classification. Primary 20–XX, 81–XX, 11F22; Secondary 20D08, 20H10, 20G45, 80R10, 81T40.

Library of Congress Cataloging-in-Publication Data

AMS-IMS-SIAM Summer Research Conference on Moonshine, the Monster, and Related Topics (1994: Mount Holyoke College)

Moonshine, the monster, and related topics : AMS-IMS-SIAM Summer Research Conference on Moonshine, the Monster, and Related Topics, June 18–23, 1994, Mount Holyoke College, S. Hadley, MA / Chongying Dong, Geoffrey Mason, editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132; v. 193)

Includes bibliographical references.

ISBN 0-8218-0385-9 (alk. paper)

1. Quantum field theory—Congresses.2. Vertex operator algebras— Congresses.3. Mathematical physics—Congresses.1. Dong, Chongying, 1958–III. Mason, Geoffrey, 1948–III. Title.IV. Series: Contemporary mathematics (American Mathematical Society); v. 193.QC174.45.A1A151994530.1'43—dc2095-40534

95-40534 CIP

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Assistant to the Publisher, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to reprint-permission@math.ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© Copyright 1996 by the American Mathematical Society. All rights reserved. The American Mathematical Society retains all rights except those granted to the United States Government.

Printed in the United States of America.

 \bigotimes The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.

C Printed on recycled paper.

10 9 8 7 6 5 4 3 2 1 01 00 99 98 97 96

Contents

Preface	ix
Higher genus Moonshine P. BÁNTAY	1
Superstring twisted conformal field theory L. DOLAN	9
Some twisted sectors for the Moonshine Module CHONGYING DONG, HAISHENG LI, AND GEOFFREY MASON	25
Spinor construction of the $c = 1/2$ minimal model ALEX J. FEINGOLD, JOHN F. X. RIES, AND MICHAEL D. WEINER	45
The McKay-Thompson series associated with the irreducible characters of the Monster KOICHIRO HARADA AND MONG LUNG LANG	93
Some quilts for the Mathieu groups TIM HSU	113
A nonmeromorphic extension of the Moonshine Module vertex operator algebra YI-ZHI HUANG	123
On the Buekenhout-Fischer geometry of the Monster A. A. IVANOV	149
Operads of moduli spaces and algebraic structures in conformal field theory TAKASHI KIMURA	159
On Hopf algebras and the elimination theorem for free Lie algebras J. LEPOWSKY AND R. L. WILSON	191
Local systems of twisted vertex operators, vertex operator superalgebras and twisted modules HAI-SHENG LI	203
Modular forms and topology KEFENG LIU	237

CONTENTS

On modular invariance of completely replicable functions YVES MARTIN	263
On embedding of integrable highest weight modules of affine Lie algebras KAILASH C. MISRA	287
The monster algebra: Some new formulae S. NORTON	297
Modular Moonshine? A. J. E. Ryba	307
Replicant powers for higher genera GENE WARD SMITH	337
Generalised Moonshine and abelian orbifold constructions MICHAEL P. TUITE	353

viii

Preface

One of the great legacies of the classification of the finite simple groups is the existence of the Monster. It was the study of this group that first suggested that there might be interesting relations between finite groups and certain elliptic modular functions, and it was this possibility—fuelled by the Conway-Norton conjectures—that led to what one might call the first version of "Moonshine", that is, the study of class functions on groups with values in a ring of modular functions.

Work of Borcherds and Frenkel-Lepowsky-Meurman led to the notion of a vertex (operator) algebra, which was seen to be the same as the chiral algebras used by physicists in conformal field theory. Nowadays one considers the Monster as the group of automorphisms of a certain vertex operator algebra—the so-called Moonshine Module—and "Moonshine" may be construed as the representation theory of certain vertex operator algebras, namely so-called orbifolds.

The connections with physics have proven to be invaluable, and it seems likely that another branch of mathematics whose origins are early similar to those of moonshine—that is, elliptic cohomology—will turn out to be very relevant too.

Most of the talks at the Moonshine Conference were devoted to one or more of these subjects, as are the accompanying papers in this volume.

Way back in 1980 in the Proceedings of the Santa Cruz conference on finite groups, when Moonshine was just emerging, Andy Ogg wrote, "... (we) should rejoice at the emergence of a new subject...rich and deep, with all the theorems yet to be proved." His remarks are as true today as they were then.

Other Titles in This Series

(Continued from the front of this publication)

- 166 George E. Andrews, David M. Bressoud, and L. Alayne Parson, Editors, The Rademacher legacy to mathematics, 1994
- 165 **Barry Mazur and Glenn Stevens, Editors,** *p*-adic monodromy and the Birch and Swinnerton-Dyer conjecture, 1994
- 164 Cameron Gordon, Yoav Moriah, and Bronislaw Wajnryb, Editors, Geometric topology, 1994
- 163 Zhong-Ci Shi and Chung-Chun Yang, Editors, Computational mathematics in China, 1994
- 162 Ciro Ciliberto, E. Laura Livorni, and Andrew J. Sommese, Editors, Classification of algebraic varieties, 1994
- 161 Paul A. Schweitzer, S. J., Steven Hurder, Nathan Moreira dos Santos, and José Luis Arraut, Editors, Differential topology, foliations, and group actions, 1994
- 160 Niky Kamran and Peter J. Olver, Editors, Lie algebras, cohomology, and new applications to quantum mechanics, 1994
- 159 William J. Heinzer, Craig L. Huneke, and Judith D. Sally, Editors, Commutative algebra: Syzygies, multiplicities, and birational algebra, 1994
- 158 Eric M. Friedlander and Mark E. Mahowald, Editors, Topology and representation theory, 1994
- 157 Alfio Quarteroni, Jacques Periaux, Yuri A. Kuznetsov, and Olof B. Widlund, Editors, Domain decomposition methods in science and engineering, 1994
- 156 **Steven R. Givant,** The structure of relation algebras generated by relativizations, 1994
- 155 William B. Jacob, Tsit-Yuen Lam, and Robert O. Robson, Editors, Recent advances in real algebraic geometry and quadratic forms, 1994
- 154 Michael Eastwood, Joseph Wolf, and Roger Zierau, Editors, The Penrose transform and analytic cohomology in representation theory, 1993
- 153 Richard S. Elman, Murray M. Schacher, and V. S. Varadarajan, Editors, Linear algebraic groups and their representations, 1993
- 152 Christopher K. McCord, Editor, Nielsen theory and dynamical systems, 1993
- 151 Matatyahu Rubin, The reconstruction of trees from their automorphism groups, 1993
- 150 Carl-Friedrich Bödigheimer and Richard M. Hain, Editors, Mapping class groups and moduli spaces of Riemann surfaces, 1993
- 149 Harry Cohn, Editor, Doeblin and modern probability, 1993
- 148 Jeffrey Fox and Peter Haskell, Editors, Index theory and operator algebras, 1993
- 147 Neil Robertson and Paul Seymour, Editors, Graph structure theory, 1993
- 146 Martin C. Tangora, Editor, Algebraic topology, 1993
- 145 Jeffrey Adams, Rebecca Herb, Stephen Kudla, Jian-Shu Li, Ron Lipsman, and Jonathan Rosenberg, Editors, Representation theory of groups and algebras, 1993
- 144 Bor-Luh Lin and William B. Johnson, Editors, Banach spaces, 1993
- 143 Marvin Knopp and Mark Sheingorn, Editors, A tribute to Emil Grosswald: Number theory and related analysis, 1993
- 142 Chung-Chun Yang and Sheng Gong, Editors, Several complex variables in China, 1993
- 141 A. Y. Cheer and C. P. van Dam, Editors, Fluid dynamics in biology, 1993
- 140 Eric L. Grinberg, Editor, Geometric analysis, 1992
- 139 Vinay Deodhar, Editor, Kazhdan-Lusztig theory and related topics, 1992

(See the AMS catalog for earlier titles)

Moonshine, the Monster, and Related Topics Chongying Dong and Geoffrey Mason, Editors

"One of the great legacies of the classification of the finite simple groups is the existence of the Monster...Work of Borcherds and Frenkel-Lepowsky-Meurman led to the notion of a vertex (operator) algebra, which was seen to be the same as the chiral algebras used by physicists in conformal field theory...The connections with physics have proven to be invaluable, and it seems likely that another branch of mathematics whose origins are eerily similar to those of moonshine—that is, elliptic cohomology—will turn out to be very relevant too." —from the Preface

This volume contains the proceedings of a Joint Summer Research Conference held at Mount Holyoke College in June 1994. As perhaps the first conference proceedings devoted exclusively to the subject known as "Moonshine", this work contains something for many mathematicians and physicists.

Features:

- Results concerning the monster simple group and other simple groups.
- Connections with elliptic cohomology.
- Connections with 2-dimensional conformal field theory.
- The role of operads.
- Connections with modular functions.

Much of *Moonshine, the Monster, and Related Topics* features new results not available anywhere else.

ISBN 0-8218-0385-9

