The Group Fixed by a Family of Injective Endomorphisms of a Free Group

Warren Dicks
Enric Ventura

Selected Titles in This Series

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by a Family of
Injective Endomorphisms
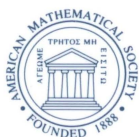of a Free Group
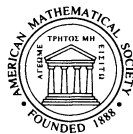
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ABSTRACT. Let F be a finitely generated free group, let ϕ be an injective endomorphism of F, and let S be a family of injective endomorphisms of F.

M. Bestvina and M. Handel proved that the rank of the free subgroup Fix(ϕ), which consists of the elements of F fixed by ϕ, is at most the rank of F. By combining their results with graph pullback techniques of J. R. Stallings, we show that, for any subgroup H of F, the rank of the intersection $H \cap \mathrm{Fix}(\phi)$ is at most the rank of H. We deduce that the rank of the free subgroup Fix(S), which consists of the elements of F fixed by every element of S, is at most the rank of F.

The topological proof given by Bestvina-Handel is here translated into the language of groupoids, and many details heretofore left to the reader are meticulously verified.

Dedicated to the memory of our late colleague and friend
Andreu Pitarch i Ribas
(1961-1993)

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This monograph contains a proof of the Bestvina-Handel Theorem (for any automorphism of a free group of rank n, the fixed group has rank at most n) that to date has not been available in book form. The account is self-contained, simplified, purely algebraic, and extends the results to an arbitrary family of injective endomorphisms.

Let F be a finitely generated free group, let ϕ be an injective endomorphism of F, and let S be a family of injective endomorphisms of F. By using the Bestvina-Handel argument with graph pullback techniques of J. R. Stallings, the authors show that, for any subgroup H of F, the rank of the intersection $H \cap \mathrm{Fix}(\phi)$ is at most the rank of H. They deduce that the rank of the free subgroup which consists of the elements of F fixed by every element of S, is at most the rank of F.

The topological proof by Bestvina-Handel is translated into the language of groupoids and many details previously left to the reader are meticulously verified in this text.