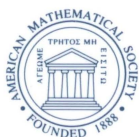


# CONTEMPORARY MATHEMATICS

195

## The Group Fixed by a Family of Injective Endomorphisms of a Free Group

Warren Dicks  
Eric Ventura



American Mathematical Society

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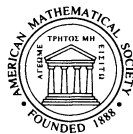
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Secondary 20E08.

**ABSTRACT.** Let  $F$  be a finitely generated free group, let  $\phi$  be an injective endomorphism of  $F$ , and let  $S$  be a family of injective endomorphisms of  $F$ .

M. Bestvina and M. Handel proved that the rank of the free subgroup  $\text{Fix}(\phi)$ , which consists of the elements of  $F$  fixed by  $\phi$ , is at most the rank of  $F$ . By combining their results with graph pullback techniques of J. R. Stallings, we show that, for any subgroup  $H$  of  $F$ , the rank of the intersection  $H \cap \text{Fix}(\phi)$  is at most the rank of  $H$ . We deduce that the rank of the free subgroup  $\text{Fix}(S)$ , which consists of the elements of  $F$  fixed by every element of  $S$ , is at most the rank of  $F$ .

The topological proof given by Bestvina-Handel is here translated into the language of groupoids, and many details heretofore left to the reader are meticulously verified.

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*Dedicated to the memory of our late colleague and friend  
Andreu Pitarch i Ribas  
(1961-1993)*

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**The Group Fixed by a Family  
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Warren Dicks and Enric Ventura

This monograph contains a proof of the Bestvina-Handel Theorem (for any automorphism of a free group of rank  $n$ , the fixed group has rank at most  $n$ ) that to date has not been available in book form. The account is self-contained, simplified, purely algebraic, and extends the results to an arbitrary family of injective endomorphisms.

Let  $F$  be a finitely generated free group, let  $\phi$  be an injective endomorphism of  $F$ , and let  $S$  be a family of injective endomorphisms of  $F$ . By using the Bestvina-Handel argument with graph pullback techniques of J. R. Stallings, the authors show that, for any subgroup  $H$  of  $F$ , the rank of the intersection  $H \cap \text{Fix}(\phi)$  is at most the rank of  $H$ . They deduce that the rank of the free subgroup which consists of the elements of  $F$  fixed by every element of  $S$ , is at most the rank of  $F$ .

The topological proof by Bestvina-Handel is translated into the language of groupoids and many details previously left to the reader are meticulously verified in this text.

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