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Operads: Proceedings of Renaissance Conferences

Special Session and International Conference on
Moduli Spaces, Operads, and Representation Theory/
Operads and Homotopy Algebra
March 1995 / May–June 1995
Hartford, Connecticut/Luminy, France

Jean-Louis Loday
James D. Stasheff
Alexander A. Voronov
Editors

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Preface

"Operads" are mathematical devices which model many sorts of algebras (such as associative, commutative, Lie, Poisson, alternative, Leibniz, etc., including those defined up to homotopy, such as A_∞-algebras). The notion of an operad appeared in the seventies in algebraic topology (J. Stasheff, J. P. May, J. M. Boardman, R. M. Vogt), but there has been a renaissance of this theory due to the discovery of relationships with graph cohomology, Koszul duality, representation theory, combinatorics, cyclic cohomology, moduli spaces, knot theory, and quantum field theory.

This renaissance was recognized at a special session "Moduli spaces, operads, and representation theory" of the AMS meeting at Hartford, CT, on March 4–5, 1995, and at a conference "Opérades et algèbre homotopique" held at the Centre International de Rencontres Mathématiques at Luminy, France, from May 29 to June 2, 1995. Both meetings drew a diverse group of researchers, as we hope these proceedings reflect, though not all the speakers are represented here.

We have arranged the contributions so as to emphasize certain themes around which the renaissance of operads took place: homotopy algebra, algebraic topology, polyhedra and combinatorics, and applications to physics. We begin the collection with two short papers which drop out of this classification. The first one is the paper "Definitions: operads, algebras and modules" by Peter May, which recalls basic notions of operad theory. The other paper, "The pre-history of operads", by one of us describes how operads had been used before they were created.

Jean-Louis Loday, Jim Stasheff, and Alexander A. Voronov

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