CONTEMPORARY MATHEMATICS

203

Geometry and Nature

In Memory of W. K. Clifford

A Conference on New Trends in
Geometrical and Topological Methods
in Memory of William Kingdon Clifford
July 30–August 5, 1995
Madeira, Portugal

Hanna Nencka
Jean-Pierre Bourguignon
Editors

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The conference on New Trends in Geometrical and Topological Methods in Memory of W. K. Clifford was held at the University of Madeira, Madeira, Portugal, from July 30–August 5, 1995. Support was provided by Centro de Ciencia e Tecnologia da Madeira, Junta Nacional de Investigacao Cientifica e Tecnologica, Fundacao Luso-Americana para o Desenvolvimento, Fundacao Jose Berardo, British Council, Camara Municipal do Funchal, Banco Internacional do Funchal, Centro de Ciencias Matematicas, Caixa Geral de Depositos, and Secretaria Regional de Turismo e Cultura.

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Preface

This issue of Contemporary Mathematics series is an outgrowth of a Conference "New Trends in Geometrical and Topological Methods" devoted to the memory of William Kingdon Clifford (Exeter 1845-Madeira 1879). The conference took place at the University of Madeira (Portugal) in July/August 1995. It's aim was to: - bring together and to put in close and informal contact active workers in fields linked to the Clifford's work, - foster interchanges of ideas and discussions between mathematicians and theoretical physicists. The meeting was divided into six one-day sessions, each one devoted to a specific aspect of Clifford's work. This volume is an attempt to present to a larger community of mathematicians and physicists, the Clifford legacy in new perspectives. The new concepts, ideas and results provided by Clifford's work are presented in this issue. This volume contains papers presented or submitted to the Conference. Each contribution is self-contained. The volume is divided in the following chapters: the first one is devoted to Clifford Algebras, the second one to Riemannian Surfaces, the third one to Information Geometry, the fourth one to Non Commutative Geometry, the fifth one to Symplectic Geometry and Self-similar Structures, the sixth one to Cosmology and General Relativity, the seventh one to Field Theory.

It is obvious that this ordering should not to be taken too seriously since in many cases there is an interplay of subjects.

This conference was made possible thanks to the support of several organizations and associations, and to the help of many friends. I am grateful to the Madeira' s Government, Dr. Francisco Santos (regional secretary of education), and L. Mendonca (President of the Madeira's Parliament). I would like to express my gratitude to the members of the Scientific and Organizing Committees, to the members of the Centre de Physique Theorique CNRS, Marseille, Luminy (France). I am grateful to the University of Madeira for all the help I had during the organization of the conference. Specially, I would like to thank the President of the Installation Committee of University of Madeira Prof. Joao David Pinto Correia, Prof. Jose Manuel Castanhera, (Vowel of I.C.U.Ma), Dr. Ana Isabel Spranger (advisor of the President of I.C.U.Ma), and Rui Almeida (Department of Physics). I wish to acknowledge the support

from CITMA and its President Dr. Maximiano Martins and all the members of CITMA's Installation Committee. I am grateful to JNICT, FLAD, Fundacao Jose Berardo, British Council, Camara Municipal do Funchal, BANIF, Centro de Ciencias Matematicas, Caixa Geral de Depositos and Secretaria Regional de Turismo e Cultura for their financial support. I would like to express my deepest and sincere gratitude to Profs. Irwin Kra and William Abikoff for the advice and to Prof. Philippe Combe and Dr. Jean-Christophe Ducom for all their help during the conference and for editing this volume.

It was with great sadness that we learned of the untimely death of Prof. Birger Iversen shortly after this conference. His contributions to the field were highly valued, and he will be greatly missed by friends and colleagues.

Hanna Nencka, Madeira , July 1996

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W.K. Clifford as a geometer

William Kingdon Clifford was born on May 4th, 1845 in Exeter, England. He received his early education at Mr.Templeton's school in Exeter. From this school he proceeded, in 1860, to King's College, London. Having in 1863 obtained a minor scholarship, he entered the Trinity College, Cambridge. In 1868 he was elected a fellow of the Trinity College and in 1871 he became Professor of Applied Mathematics and Mechanics at the University College, London, a position he occupied until his death.

In June, 1874, Clifford was elected a Fellow of the Royal Society. On June 18th, 1866, he became a very active member of the London Mathematical Society, and served on its Council for every session from 1868-1869 to 1876-1877.

Clifford was above all a geometer, his mathematical writings can be for a large part classified as geometry[1]. In the treatment of geometrical questions he showed a notorious preference for symbolical methods, and as might be expected, a marvellous command over analytical expression. He took much pleasure in speculative constructions of complex or non-Euclidian systems of space-relations. He had a very strong geometrical imagination. The properties of space remained a perpetual subject of contemplation for him. Purely analytical considerations undertaken by W.K. Clifford without any impulse from reference to geometry are few. His most important works are devoted to Riemann's surfaces, biquaternions, and classification of loci.

Clifford attributed to geometry the widest imaginable scope. He was a mathematician as well as a metaphysician; and geometry was to him an important factor in the problem of "describing the universe". In his lecture *"On the postulates of the Science of Space"* he stated his own views on this question. The pages in which he expressed them are likely to be remembered, as marking an important moment in the controversy concerning the nature of space and the origin of our knowledge of it, which is likely to last as long as metaphysical enquiries have any interest for mankind. In his lectures he enumerated four fundamental postulates on which the ordinary conception of space is founded: 1 - its

[1]see e.g. Mathematical Papers by William Kingdon Clifford, Edited by Robert Tucker, Chelsea Publishing Compagny, Bronx, New York.

continuity, 2 - its flatness in its smallest part, 3 - its similarity to itself at every point, and 4 - the possibility of the existence of figures similar to one another, but on different scales of magnitude.

The second postulate consists of one of his favourite hypothesis and has been described in the communication *"On the Space-Theory of Matter"* at the Cambridge Philosophical Society, in 1870, where he developed the idea that space may not be independent of time. This idea was explicitly stated in a book: *"Common Sense of the Exact Sciences"* published by Rowe after his death, where he wrote:

We may conceive our space to have everywhere a nearly uniform curvature, but that slight variations of the curvature may occur from point to point, and themselves vary with time. These variations of the curvature with the time may produce effects which we not unnaturally attribute to physical causes independant of the geometry of our space. We may even go as far as to assign to this variation of the curvature what really happens in that phenomenon which we term the motion of matter.

The study of the geometrical methods of Grassmann so long neglected, made a great and enduring impression on Clifford. In his paper (1876) *"Applications of Grassmann's Extensive Algebra"*, speaking about the work of Grassmann he had the prophetic sentence:

I may be permitted to express my profund admiration of that extraordinary work, and my conviction that its principles will exercise a vast influence upon the future of mathematical science.

It is one more time through geometry that Clifford approached a problem in algebra to provide a newer view, and his name is perpetuated today in the so-called Clifford algebras, of which octonions or biquaternions are special cases. These noncommutative algebras were used by Clifford to study motions in non-Euclidean spaces, certain manifolds which are now known as spaces of Clifford and Klein.

In the spring of 1876, distinct and grave indications of pulmonary disease were noted, but to be careful about himself never occurred to him. In the early months of 1878 there came a sudden change for the worst. Since medicine had no new thing to recommend, and almost nothing to forbid, a last experiment could only be tried and Clifford sailed for Madeira. The change from the bitterness of recent English winters to the fair and temperate air of Madeira had unfortunately no power to restore his waning forces; but it enabled him to spend his last days in ease and comparative enjoyment. Some weeks were added to his life but on March 3, 1879, the end came.

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