CONTEMPORARY MATHEMATICS

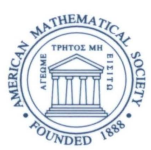
208

Harmonic Analysis and Nonlinear Differential Equations

A Volume in Honor of Victor L. Shapiro

Michel L. Lapidus, Coordinating Editor

Lawrence H. Harper
Adolfo J. Rumbos
Editors



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Harmonic Analysis and Nonlinear Differential Equations

Victor L. Shapiro

CONTEMPORARY MATHEMATICS

208

Harmonic Analysis and Nonlinear Differential Equations

A Volume in Honor of Victor L. Shapiro

November 3–5, 1995
University of California, Riverside

Michel L. Lapidus, Coordinating Editor

Lawrence H. Harper
Adolfo J. Rumbos
Editors

American Mathematical Society
Providence, Rhode Island

This volume is a collection of papers dealing with harmonic analysis and nonlinear differential equations and stems from a conference in honor of Victor L. Shapiro. This conference was held in conjunction with the fall meeting of the Southern California Analysis and Partial Differential Equations Conference, November 3–5, 1995, in Riverside, CA, and was jointly sponsored by the National Science Foundation and the University of California, Riverside.

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Dedication

This volume is dedicated to Victor L. Shapiro in recognition of his long and productive mathematical career, particularly his many valuable contributions to harmonic analysis and quasilinear partial differential equations.

Victor Lenard Shapiro was born in Chicago, Illinois, on October 16, 1924. In September 1943, he entered the United States army at the height of World War II. In September 1944, he was awarded the expert infantry badge for action against the enemy on the South Pacific island of Bougainville. In April 1945, serving as a combat medic with the 132nd infantry regiment, Vic was in the 6th wave of a beachhead landing on Cebu, Phillipine Islands, and saw much action in the ensuing campaign.

After the war, in June 1946, Vic entered the University of Chicago with the help of the G. I. Bill. He received his Ph.D. degree in June 1952. His doctoral dissertation, written under the direction of A. Zygmund, dealt with the localization theory of multiple trigonometric series.

Shapiro was subsequently a professor at Rutgers University (8 years), the University of Oregon (4 years) and the University of California, Riverside (32 years+). In between, he spent a total of 3 years as a member of the Institute for Advanced Study at Princeton, part of the time as a post-doctoral National Science Foundation fellow.

V. L. Shapiro is mainly known for having solved a number of difficult problems in analysis:

(i) He obtained a best possible result on the uniqueness of Abel sums of multiple trigonometric series, establishing as a consequence the analog of Cantor's famous uniqueness theorem for double trigonometric series under circular convergence with a side condition. The side condition was later shown to hold by R. Cooke. Zygmund himself had emphasized the significance of this problem and it had confounded some of the best analysts of the day. Recently, J. Bourgain has extended the theorem to all dimensions $d \geq 3$.

(ii) He proved the analyticity of the Cauchy-Riemann equations under pointwise L^1-total-differentials, which had also been an open problem for many years.

(iii) He proved (in collaboration with his student, R. Crittenden) an analog of Cantor's theorem for Walsh-Fourier series. This had also been a long-standing open problem.

(iv) He did fundamental work on removable singularities and exceptional sets for the Navier-Stokes equations.

(v) Later, shifting over to more general quasilinear differential equations, he wrote a fundamental paper on resonance and the first eigenvalue for such equations. His most recent work involves time-periodic boundary value problems and reaction-diffusion equations.

In 1964, Victor was given an award by the National Science Foundation for his expository paper "Fourier Series in Several Variables" which appeared in the *Bulletin of the American Mathematical Society.* In 1978, Victor was elected the Faculty Research Lecturer at the University of California, Riverside (UCR). In his lecture, entitled "Exceptional Sets and the History of Mathematics," he showed how the apparently arcane technicalities of exceptional sets for trigonometric series had led Cantor to the basic results of set theory, thereby profoundly influencing the course of mathematics.

Victor has also been recognized as an outstanding teacher at the upper division and graduate level, mainly teaching courses on applied mathematics. Bill Wade, one of the 20 students who received their Ph.D. degrees under Victor's tutelage, began his Shapirofest lecture with the following remarks about Vic as a teacher:

"Vic's style was a revelation to me. Before (taking his class) I was convinced that algebra was the only rigorous branch of mathematics, and I was going to be an algebraist. But Vic put in every detail. There was no hand waving about orders of growth, no vague phrases about "as small as you like," no appealing to mysterious principles from physics – just good solid mathematics based on inequalities and estimates. The estimates were carried out in precise, complete detail, in steps which were easy to follow and could not be refuted. Moreover, he shared enough of the motivation behind the results that we could almost anticipate the next result he planned to prove. All of a sudden I realized that analysis was an elegant, natural, almost simple branch of mathematics. And I decided I wanted to work in this area for the rest of my life."

Altogether, Vic, and the doctoral theses of the students working under him, have accounted for 18 papers published in the *Transactions of the American Mathematical Society.*

On his own initiative, Vic created a course at UCR on the History of Mathematics and taught it successfully for a number of years. Recently he did the same for Mathematical Biology. His hobbies are ballroom dancing with his wife, Florence, and hiking with their four children and eight grandchildren.

Because of his distinguished research work, inspirational teaching and exemplary humanity, we, as his students, colleagues and collaborators, dedicate this volume to Victor L. Shapiro.

Lawrence H. Harper, Michel L. Lapidus and Adolfo J. Rumbos

Preface

This volume is a collection of papers dealing with harmonic analysis and nonlinear differential equations and stems from a conference in honor of Victor L. Shapiro specifically on these two mathematical areas and their interface. This conference was held in conjunction with the fall meeting of the Southern California Analysis and Partial Differential Equations Conference, November 3–5, 1995, in Riverside, California, and was jointly sponsored by the National Science Foundation and the University of California, Riverside.

Most of the papers presented in this book are from individuals who spoke at the conference and are closely related to the various areas of analysis in which Vic Shapiro has worked during the last four and one-half decades.

In particular, there are four papers dealing directly with the interface between harmonic analysis and nonlinear partial differential equations, one on the d-torus and the nonlinear Schrödinger equation, two using harmonic analysis to study periodic solutions of the Navier-Stokes equations, one of which has applications to turbulence theory, and a fourth dealing with double Fourier series and time-periodic solutions of the semilinear wave equation. Furthermore, there are two up-to-date overviews of the recent developments in two important branches of harmonic analysis, namely one dealing with the latest spectacular results in the uniqueness theory of multiple trigonometric series and another covering the current work in dyadic harmonic analysis (i.e., Walsh-Fourier series). In addition, there is another overview of important recent work on shock-waves, black holes, and general relativity, as well as a survey (and research) article of recent work on harmonic analysis and PDEs on fractals, and their relationships with noncommutative geometry, of relevance, for example, in condensed matter physics, particularly the study of diffusions and wave propagation in porous media or percolation networks.

There also are several research articles that deal primarily with nonlinear differential equations, one using variational methods to solve higher order quasilinear elliptic equations, another concerned with new results in dynamical systems which have applications to neural networks, a third dealing with the structure of the solution set of semilinear equations involving the Laplacian in $\mathbb{R}^n$, and a fourth concerned with an abstract evolution equation with applications to the blow up of solutions to the nonlinear wave equation. There is another paper in this volume containing recent historical work on the Mary Cartwright-J. E. Littlewood collaboration on the Van der Pol equation, as well as one that deals with the application of dyadic harmonic analysis to coding theory. There is also a paper which gives an overview of recent work in the q-theory of special functions with applications to some classical trigonometric expansions.

A number of the above papers contain open problems in their respective areas.

In closing this foreword, the editors would like to express their appreciation to a number of individuals and sponsors who made the conference and this volume possible.

First, we would like to thank Professor Albert R. Stralka, former Chair of the Department of Mathematics at UCR, for his initial suggestion to organize this symposium in honor of Professor Shapiro. We are also grateful to Professors M. Salah Baouendi and Linda Preiss Rothschild for their advice during the initial preparation of the conference. Further, we wish to thank Cheryl Ann Griffith, Harry Watson, Jr. and Duane Allen for generously donating their time during the conference, as well as Flo Shapiro for her never failing optimism and her help in setting-up the banquet in honor of her husband. Our thanks also go to Jan Patterson for typing-up parts of the manuscripts, to Julie Drouyor for her help in collecting the manuscripts and to Linda Terry for her assistance in planning the conference. The coordinating editor (M. L. L.) is grateful to Professor Donald G. Babbitt, Director of Publications at the American Mathematical Society, for a long phone conversation regarding the possible publication of this volume in *Contemporary Mathematics*, shortly after the conference was held.

Finally, we wish to acknowledge the financial support of the National Science Foundation and the University of California, Riverside (UCR), more specifically, the Department of Mathematics and the Dean of the College of Natural Sciences and Agriculture, as well as the Jones' Chair of Mathematics at UCR.

Michel L. Lapidus, Lawrence H. Harper and Adolfo J. Rumbos

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Harmonic Analysis and Nonlinear Differential Equations: A Volume in Honor of Victor L. Shapiro

Michel L. Lapidus, Lawrence H. Harper, and Adolfo J. Rumbos, Editors

This volume is a collection of papers dealing with harmonic analysis and nonlinear differential equations and stems from a conference on these two areas and their interface held in November 1995 at the University of California, Riverside, in honor of V. L. Shapiro. There are four papers dealing directly with the use of harmonic analysis techniques to solve challenging problems in nonlinear partial differential equations. There are also several survey articles on recent developments in multiple trigonometric series, dyadic harmonic analysis, special functions, analysis on fractals, and shock waves, as well as papers with new results in nonlinear differential equations. These survey articles, along with several of the research articles, cover a wide variety of applications such as turbulence, general relativity and black holes, neural networks, and diffusion and wave propagation in porous media. A number of the papers contain open problems in their respective areas.

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