CONTEMPORARY MATHEMATICS

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Operator Theory for Complex and Hypercomplex Analysis

Operator Theory for Complex and Hypercomplex Analysis December 12–17, 1994 Mexico City, Mexico

E. Ramírez de Arellano N. Salinas M. V. Shapiro N. L. Vasilevski Editors



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This volume contains the proceedings of an international conference that took place in Mexico City, December 12–17, 1994. The subject of the conference was chosen to highlight the interplay between operator theory and complex and hypercomplex analysis as well as to study some classes of operators appearing in them.

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PREFACE

Integral representations for different classes of functions in analysis motivate the introduction and study of a series of important operators: singular integral, Toeplitz, Bergman, convolution operators on Lie groups, some classes of pseudodifferential operators, etc. Investigation of these operators develops and enriches "pure" operator theory and exerts a stimulating influence on important areas of analysis.

This volume contains the proceedings of an international conference that took place in Mexico City, December 12–17, 1994. The subject of the conference was chosen to highlight the interplay between operator theory and complex and hypercomplex analysis as well as to study some classes of operators appearing in them.

The editors wish to thank the contributors to this volume and also the referees for their many helpful comments to the authors. They wish to apologize for the unfortunate delay of its publication but it was not easy to interact through the pitfalls of communication among different countries, computer systems, and various incompatible codes. The task of the editors was made not only possible but infinitely much easier by the cooperation and technical assistance of Mrs. Larisa Martin, who retyped some of the manuscripts in the appropriate form, formatted all the manuscripts according to the Contemporary Math. Journal specifications, and put together the final version of the volume for publication. The editors are indebted to her.

The conference was made possible thanks to the generous support of the Center for Research and Advanced Study (CINVESTAV-IPN) and of the School of Physics and Mathematics of the National Polytechnic Institute (ESFM-IPN). Special thanks is due to the Consejo Nacional de Ciencia y Tecnología (CONACYT-México) and to the National Science Foundation (NSF-USA) for their financial support.

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Operator Theory for Complex and Hypercomplex Analysis

E. Ramírez de Arellano, N. Salinas, M. V. Shapiro, and N. L. Vasilevski, Editors

This book presents a collection of papers on certain aspects of general operator theory related to classes of important operators: singular integral, Toeplitz and Bergman operators, convolution operators on Lie groups, pseudodifferential operators, etc. The study of these operators arises from integral representations for different classes of functions, enriches pure operator theory, and is influential and beneficial for important areas of analysis. Particular attention is paid to the fruitful interplay of recent developments of complex and hypercomplex analysis on one side and to operator theory on the other. The majority of papers illustrate this interplay as well as related applications. The papers represent the proceedings of the conference "Operator Theory for Complex and Hypercomplex Analysis", held in December 1994 in Mexico City.

