CONTEMPORARY MATHEMATICS

219

Secondary Calculus and Cohomological Physics

Proceedings of a Conference on Secondary Calculus and Cohomological Physics August 24–31, 1997 Moscow, Russia

> Marc Henneaux Joseph Krasil'shchik Alexandre Vinogradov Editors



American Mathematical Society

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Secondary Calculus and Cohomological Physics

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Proceedings of a Conference on Secondary Calculus and Cohomological Physics August 24–31, 1997 Moscow, Russia

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ACKNOWLEDGMENTS

The International Conference Secondary Calculus and Cohomological Physics was organized by an initiative of the DIFFIETY INSTITUTE of the Russian Academy of Natural Sciences and in collaboration with the M.V. LOMONOSOV MOSCOW STATE UNIVERSITY and ISTITUTO ITALIANO PER GLI STUDI FILOSOFICI. It became possible thanks to the financial support of

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Foreword

It is common knowledge that the mainstream in the theory of partial differential equations went along analytical lines in the twentieth century. The geometrical approach originating from classical work by S. Lie and B.V. Bäcklund kept a kind of a marginal position for a long time, and maybe this is one of the reasons for the words of capitulation pronounced by R. Courant: "problems related with differential equations are so highly diversified that the general theory can be hardly constructed"¹. Meanwhile, Quantum Field Theory and its generalizations underwent remarkable developments. This progress on the physics side was done with little input from the geometrical theory of partial differential equations and gave birth to numerous fantastic creatures — ghosts and antighosts being the most civil in this bestiary. This absence of input is perhaps explained by a proverbial remark ascribed to J. Gibbs: "mathematics is a language". In other words, physicists did not find mathematical terms to express adequately their field theoretical concepts. And this is completely due to mathematicians' negligence in this respect.

Fortunately, in recent years the situation has begun to change. On one hand, developments started with the famous BRST-transformation have demonstrated a deep cohomological nature of some key constructions in Quantum Field Theory. On the other hand, adequate mathematical tools can be found in the modern geometric and algebraic theory of nonlinear partial differential equations, which results from the above-mentioned work by Lie and Bäcklund and deals with infinite jets, prolongations, invariant algebraic constructions, etc.

This conference was held to promote the confluence of these two streams, i.e., of *Cohomological Physics* and *Secondary Calculus*, which appears to be an extremely promising development. More exactly, the term Secondary Calculus refers to topics such as higher symmetries of PDE and the C-spectral sequence (variational bicomplex), which are secondary analogues of usual vector fields and differential forms. All other ingredients of the Classical Calculus have the corresponding secondary counterparts and the BRST-transformation, the Batalin–Vilkovisky approach, and anti-fields formalism, among others.

Papers composing these Proceedings were written by both mathematicians and physicists. Even a superficial look shows a deep affinity between them. For instance, anti-fields can be easily identified with secondary first order differential forms on the infinite jet space, and the ghosts materialize in a rather rational way. So we hope that this collection will stimulate a mathematician to develop Secondary Calculus to cover the needs of QFT and its generalizations much better, and a physicist to make more efficient use of results and methods coming from it.

* * *

¹R. Courant, Partial Differential Equations, New York (1962).

FOREWORD

The meeting took place August 24-31, 1997, at the internationally known Main Building of the Moscow State University and brought togerther over sixty participants from Europe, North America and Japan. The program included 50-minute invited lectures and 35- and 20-minute lectures and communications in a strong mixture of mathematical and physical talks (see the list below). The participants found this experiment successful and rather stimulating. Almost all invited lectures are published in these Proceedings, while a part of the communications is stored at the Electronic Publishing House WWW-site (http://www.ephouse.com/math/).

The list of invited speakers included:

M. Asorey, G. Barnich, C. Becchi, L. Bonora, F. Brandt, R. Bryant, M. Fliess, M. Henneaux, V. Kozlov, J. Krasil'shchik, A. Ibort, N. Maggiore, G. Marmo, M. Rasetti, J. Stasheff, A. Verbovetsky, C. Viallet, and A. Vinogradov (together with I. Anderson, M. Dubois-Violette, P. Griffiths, and N. Kamran, who, for various reasons, were not able to attend).

The complete list of the contributions to the Conference can be found below.

Marc HENNEAUX Joseph KRASIL'SHCHIK Alexandre VINOGRADOV

Brussels-Moscow-Salerno

LIST OF LECTURES AND COMMUNICATIONS

Vladimir ANDREEV (Russia), Infinite dimensional algebras connected with matrixes integrable equations.

Manuel ASOREY (Spain), Topological transitions.

Glenn BARNICH (Belgium), Sh Lie structure of Poisson brackets in field theory.

Carlo BECCHI (Italy), BRS cohomology in topological field theory.

Loriano BONORA (Italy), Review of BRST in superstring theory.

Friedemann BRANDT (Spain), Gauge covariant algebras and local BRST cohomology.

Robert BRYANT (USA), Conservation laws and the geometry of PDE.

Marco CASTRILLON (Spain), Gauge invariance on principal SU(2)-bundles.

Vladimir CHETVERIKOV (Russia), Symmetries of some integro-differential equations.

Alexei DAVYDOV (Russia), On some class in the Hochschild cohomology of fusion algebras.

Michel FLIESS (France), Control theory and infinite jets: applications and introductory theory.

Kenji FUKAYA (Japan), Homological/homotopical algebra, topological field theory, and moduli spaces.

Yuri GLIKLICH (Russia), On stochastic evolution of a general relativistic quantum particle without conservation of energy.

Soso GOGILIDZE, Arsen KHVEDELIDZE (Russia), First integrals and Abelianization of first class constraints.

Dmitry GUREVICH (France), Differential complexes associated to the QYBE.

Marc HENNEAUX (Belgium), Consistent interactions between gauge fields: the cohomological approach.

Alberto IBORT (Spain), Equivariant Floer's cohomology.

Tatiana IVANOVA (Russia), Čech cohomology and symmetries of the self-dual Yang – Mills equations.

Ol'ga IVANOVA (Russia), Reduced system of Bethe ansatz equations and geometry of the parametric surfaces.

Igor KANATCHIKOV (Estonia), On the noncommutative Gerstenhaber algebra of the covariant Poisson brackets on forms in field theory.

Paul KERSTEN (the Netherlands), Super WDVV-equations and solutions in (3,2) and (3,4) dimensions.

Nina KHOR'KOVA (Russia), On reconstruction problem for nonlocal symmetries.

Hovhannes KHUDAVERDYAN (Russia), Odd invariant semidensity and divergence-like operators on an odd symplectic superspace.

Valery KOZLOV, Sergei BOLOTIN (Russia), Symmetry and topology of dynamical systems.

Joseph KRASIL'SHCHIK (Russia), Cohomological background for recursion operators and Poisson structures in PDE.

Andrei KUDRYAVTSEV (Russia), On symmetries and conservation laws for the generalized Khokhlov – Zabolotskaya equations.

Ol'ga KUNAKOVSKAYA (Russia), Some functors on the category of SC^r -manifolds.

Vitaly KUSHNIREVITCH (Ukraine), Differential structures in Leibniz superalgebra.

Tom LADA (USA), SH Lie structures of gauge symmetries in higher order spin theories.

Alexandre LAVRENOV (Belarus), Symmetry of the discrete models based on Askey – Wilson algebra.

Pierre LECOMTE (Belgium), Classification of modules of differential operators (joint work with Mathonet, Ulg, and Ovsienko, CPT Marseille-Luminy).

Nicola MAGGIORE (Italy), Algebraic renormalization of (massive) supersymmetric gauge theories.

Giuseppe MARMO (Italy), Cohomology in classical dynamics.

Michal MARVAN (Czech Republic), Direct computation of zero-curvature representations.

Valentin OVSIENKO (France), Exotic deformation quantization.

Michael PARINOV (Russia), One method of obtaining first integrals of Lorentz equation.

Jean-Baptiste POMET (France), Geometry for dynamic feedback in nonlinear control systems.

Alexander POPOV (Russia), Symmetries of self-dual gravity and Kodaira-Spencer deformation theory.

Mario RASETTI, Vittorio PENNA (Italy), Physics on non-Euclidean lattices. Pedro REAL JURADO (Spain), Homological perturbation theory and computability of the homology of commutative DGA-algebras (joint work with V. Alvarez, J.A. Armario and B. Silva).

Dmitry ROYTENBERG (USA), Lie bialgebroids and strongly homotopy Lie algebras (joint work with Alan Weinstein).

Vladimir RUBTSOV (France), Separation of variables for Hitchin systems.

Alexey SAMOKHIN (Russia), Symmetries of the equations containing a small parameter.

Valery SHEMARULIN (Russia), A direct method for solving initial-value problems.

Vladimir SOKOLOV (Russia), On integrable evolution non-Abelian equations.

Vladimir SOLOVIEV (Russia), Total divergences as a grading of the formal variational calculus.

Jim STASHEFF (USA), The (secret?) homological algebra of the Batalin – Vilkovisky complex.

Yuri TORKHOV (Russia), Higher differential forms and their applications.

Anatolii TSIRLIN (Russia), Vladimir KAZAKOV (Australia), Averaged relaxations of extremal problems.

Alexandre VERBOVETSKY (Russia), Horizontal (characteristic) cohomology and compatibility complex.

Claude VIALLET (France), On the invariants of some discrete groups.

Gaetano VILASI, Giuseppe MARMO, Alexandre VINOGRADOV (Italy), On the local structure of n-Poisson and n-Jacobi manifolds.

Alexandre VINOGRADOV (Italy), An introduction to Secondary Calculus.

Alexandre VINOGRADOV (Italy), Some recent results in Secondary Calculus.

Michael VINOGRADOV (Russia), *n*-ary Lie algebras of Poisson type (joint work with A. Vinogradov).

Alexander YAKHNO (Russia), Conservation laws and main boundary problems of plasticity.

Valery YUMAGUZHIN (Russia), Conditions of reducibility of ODE's to the form y''' = 0.

Michael ZELIKIN (Russia), Solution to quadratic PDE as a connection on the integral Dirichlet's bundle.

Victor ZHARINOV (Russia), Secondary Operational Calculus.

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Secondary Calculus and Cohomological Physics

Marc Henneaux, Joseph Krasil'shchik, and Alexandre Vinogradov, Editors

This collection of invited lectures (at the Conference on Secondary Calculus and Cohomological Physics, Moscow, 1997) reflects the state-of-the-art in a new branch of mathematics and mathematical physics arising at the intersection of geometry of nonlinear differential equations, quantum field theory and cohomological algebra. This is the first comprehensive and self-contained book on modern quantum field theory in the context of cohomological methods and the geometry of nonlinear PDEs.

Features:

- an up-to-date and self-contained exposition of the newest results in cohomological aspects of quantum field theory and the geometry of PDEs
- a new look at interrelations between cohomology theory, the geometry of PDEs and field theory
- application to Batalin-Vilkovisky formalism, BRST formalism, anomalies and quantum dynamics



