

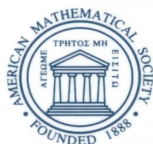
CONTEMPORARY MATHEMATICS

223

Advances in Discrete and Computational Geometry

Proceedings of the 1996 AMS-IMS-SIAM
Joint Summer Research Conference
Discrete and Computational Geometry: Ten Years Later
July 14–18, 1996
Mount Holyoke College

Bernard Chazelle
Jacob E. Goodman
Richard Pollack
Editors



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Preface

On July 14–18, 1996, Mt. Holyoke College hosted an AMS-IMS-SIAM Summer Research Conference on the theme “Discrete and Computational Geometry: Ten Years Later.” The meeting was organized by B. Chazelle, J. E. Goodman, J. O’Rourke, J. Pach, and R. Pollack. The title harked back to a 1986 conference in Santa Cruz, organized by J. E. Goodman and R. Pollack. Its contagious success heralded years of fruitful interaction between the established field of discrete geometry and the then youthful offshoot from computer science, computational geometry.

This volume presents a collection of research and survey articles, most of which were presented at the Mt. Holyoke meeting. The mix of articles in this volume touches on a wide variety of current topics in discrete and computational geometry. It includes surveys on range searching, computational topology, packing and covering, real algebraic geometry, visibility, universality theorems, tilings, arrangements, geometric discrepancy theory, transversal theory, and polytopes, as well as research contributions on monotone paths, flag complexes, discrepancy theory, geometric graph theory, polygonal covers, and Helly-type theorems.

In “Geometric range searching and its relatives,” Agarwal and Erickson provide a comprehensive survey of the vast recent literature on the computational complexity of searching in higher-dimensional space. Geometric sampling and space partitioning techniques (e.g., cuttings) are discussed in great detail.

The next article, “Deformed products and maximal shadows of polytopes,” by Amenta and Ziegler, builds on the classical result of Klee and Minty on the exponential length of pivoting sequences for the simplex algorithm. It presents sharp bounds on the maximal lengths of increasing paths and planar projections.

In “Flag complexes, labelled rooted trees, and star shellings,” Billera, Chan, and Liu establish and investigate an intriguing correspondence between flag complexes and labelled rooted trees, and they derive enumerative results concerning complexes of cubes called star-shellable.

In the following article, “Discrepancy bounds for geometric set systems with square incidence matrices,” Chazelle proves the existence of n triangles and n points such that for any two-coloring of the points there is at least one triangle which contains at least $\Omega(n^{1/4})$ more points of one color than those of the other color. The result was previously known only if the number of triangles is asymptotically greater than the number of points.

“Computational topology,” by Dey, Edelsbrunner, and Guha, provides a thorough, accessible introduction to the nascent field of computational topology: applications can be found in robotics, molecular modeling, image processing, cartography, mesh generation, etc.

In “Recent progress on packing and covering,” G. Fejes Tóth surveys the last ten years of research in the theory of packing and covering, one of the oldest and most developed areas in discrete geometry.

“Acoptic polyhedra” are polyhedra with no self-intersections. Grünbaum surveys their basic properties in 3-space. The motivation is to study how crucial a requirement convexity is in deriving standard properties of polytopes.

The article by Holt and Klee, “A proof of the strict monotone 4-step conjecture,” establishes a variant of Hirsch’s celebrated conjecture for the case of a 4-polytope with 8 facets, along with many related facts.

In “Interactions between real algebraic geometry and discrete and computational geometry,” Itenberg and Roy introduce and survey the fast-evolving field of real algebraic geometry, highlighting its discrete-geometric, combinatorial, topological, and algorithmic aspects. Many of the most recent results on combinatorial patchworking, roadmaps, and quantifier elimination can be found in this article.

The next contribution, “Open problems in the combinatorics of visibility and illumination,” is a large collection of open combinatorial and computational problems concerning art gallery theorems and related visibility questions.

“Halving lines and perfect cross-matchings,” by Pach and Solymosi, characterizes the sets of $2n$ points in the plane (in general position) having a perfect matching with pairwise crossings as being precisely those admitting n halving lines. This leads to an efficient algorithm for checking the existence of such matchings.

In “Three-dimensional grid drawings of graphs,” Pach, Thiele, and Tóth prove that any n -vertex graph can be embedded on a three-dimensional grid of volume $O(n^2)$, provided that its chromatic number is bounded.

The next article, “On polygonal covers,” by Pocchiola and Vegter, gives a constructive proof of a result by L. Fejes Tóth asserting the possibility of covering n disjoint convex bodies in the plane by polygons totaling at most $6n - 9$ edges.

In “The universality theorems for oriented matroids and polytopes,” Richter-Gebert surveys the main developments in the polyhedral theory of universality: these concern the complexity (e.g., computational, topological, algebraic) of the realization spaces for polytopes and oriented matroids.

Senéchal’s contribution, “Periodic and aperiodic tilings of E^n ,” is an accessible survey of the beautiful developments on planar tilings and, in particular, the relations between local properties and global ones (such as periodicity).

If computational geometry has a father, Shamos’s claim to the title is as strong as anyone’s. He is the one who massaged an eclectic collection of problems and curiosities into a new field of science. In “The early years of computational geometry—a personal memoir,” Shamos reminisces. His account is mostly personal, often anecdotal, always enjoyable.

The next article, “Arrangements of surfaces in higher dimensions,” by Sharir, reviews recent progress in the study of arrangements for surfaces embedded in d -space. Numerous applications can be found in motion planning, geometric optimization, and transversal theory. The results—many by the author himself—represent the culmination of over a decade of active research on this topic.

In “Geometric discrepancy theory,” Spencer gives an engaging account of a subject at the crossroads of geometry, number theory and combinatorics, with an emphasis on some of the most exciting recent developments.

The following contribution, “Proof of Reay’s conjecture on certain positive-dimensional intersections,” by Tverberg, settles a conjecture of Reay on an extension of Radon’s theorem to one-dimensional intersections.

The last ten years have witnessed an explosion of results in geometric transversal theory. In “Progress in geometric transversal theory,” Wenger walks us through its main developments, beginning at the starting point of the subject, i.e., Hadwiger’s seminal extension of Helly’s theorem to line transversals, and touching on such contemporary concepts as combinatorial complexity, oriented matroids, and piercing numbers.

The next article, “Recent progress on polytopes,” by Ziegler, offers seven open problems in the combinatorial theory of convex polytopes. The discussion centers around five main subjects: universality theorems, triangulations, 0/1 polytopes, neighborly polytopes, and monotone paths. This contribution ties in beautifully with the articles above by Richter-Gebert, Holt and Klee, and Amenta and Ziegler.

The volume ends with an appendix devoted to the Computational Geometry Impact Task Force Report. This document was prepared by a panel, chaired by Chazelle, consisting of 30-odd computer scientists and geometers. It discusses the challenges and opportunities ahead for the field of computational geometry in view of its recent accomplishments.

The editors extend their warm gratitude to the authors for their willing participation, and to the many anonymous referees whose help was indispensable in bringing this volume to fruition.

Happy reading!

Bernard Chazelle
Jacob E. Goodman
Richard Pollack

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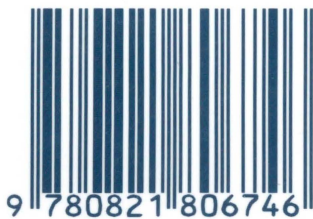
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