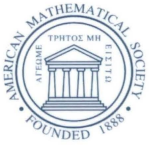
CONTEMPORARY MATHEMATICS

238

Nonlinear Partial Differential Equations

International Conference on
Nonlinear Partial Differential Equations
and Applications
March 21–24, 1998
Northwestern University

Gui-Qiang Chen
Emmanuele DiBenedetto
Editors



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Providence, Rhode Island

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This volume contains refereed original research papers and expository articles resulting from the scientific program of the 1997–98 Nonlinear PDE Emphasis Year at Northwestern University. The papers present a cross-section of the most significant recent advances and current trends and directions in nonlinear partial differential equations and related topics; most are by speakers at the International Conference on Nonlinear Partial Differential Equations, held March 21–24, 1998, in Evanston, IL.

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Preface

This volume is a collection of refereed original research papers and expository articles and stems from the scientific program of the 1997-98 Nonlinear PDE Emphasis Year at Northwestern University, which was jointly sponsored by Northwestern University and the National Science Foundation. Most of the papers presented are from the distinguished mathematicians who spoke at the International Conference on Nonlinear Partial Differential Equations, March 21-24, 1998, Evanston, IL.

The book is a cross-section of the most significant recent advances and current trends and directions in nonlinear partial differential equations and related topics. Contributions range from modern approaches to the classical theory in elliptic and parabolic equations to nonlinear hyperbolic systems of conservation laws and their numerical treatment.

The general guiding idea in editing this volume has been twofold. On one hand, we have solicited the papers that contribute in a substantial way to the general analytical treatment of the theory of nonlinear partial differential equations. On the other hand, we have attempted to collect the contributions to computational methods and applications, originating from significant realistic mathematical models of natural phenomena, to seek synergistic links between theory and modeling and computation and to underscore current research trends in partial differential equations. The borderline between these two aspects of mathematical research is rather fuzzy. We have also selected a set of papers that would bridge them.

Examples of the first kind of contributions include new insights into the role of the Harnack inequality in the theory of fully nonlinear elliptic equations, new results on the local behavior of degenerate parabolic equations, a treatment of the complex eikonal equations, and the solvability of implicit degenerate elliptic systems and motion by curvature.

Included in this broad category also are the papers establishing the regularity, large-time behavior, and L^1 stability of entropy solutions, the analysis of nonclassical shocks, and the convergence of the vanishing viscosity method for initial-boundary value problems for nonlinear hyperbolic systems of conservation laws, as well as the asymptotic stability of diffusion waves for the multidimensional nonlinear wave equations and the structural stability of steady-state solutions of the Navier-Stokes equations.

Contributions of the second kind range from numerical methods for predicting the large-scale dynamics and multidimensional simple front tracking algorithms to mathematical aspects of turbulent convection, geometry of crystal shapes, singularities in relativity and plasma dynamics, and high field kinetic semiconductor models.

This volume would not have been possible without the help and support of a number of people and institutions. First, we would like to thank the American Mathematical Society, especially, Edward G. Dunne (Editor of Book Program), Christine M. Thivierge (Acquisitions Assistant), Deborah Smith (Production Editor), and the technical support group for their prompt and professional assistance and their patience with our slow pace.

Karen Townsend deserves our special thanks for her assistance in the review process.

We are also grateful to the referees for their constructive criticisms and suggestions, and to Konstantina Trivisa and Mikhail Feldman for their invaluable help with the editorial work.

Finally, we wish to acknowledge the financial support of the National Science Foundation through grant DMS-9708261 and Northwestern University, more specially, the Department of Mathematics and the Office of the Vice President for Research of Northwestern University.

The purchaser of this volume is entitled to the online version of this book by the AMS. To gain access, follow the instructions given on the form found in the back of this volume.

Gui-Qiang Chen and Emmanuele DiBenedetto
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The book's contributions offer two perspectives. There are papers on general analytical treatment of the theory and papers on computational methods and applications originating from significant realistic mathematical models of natural phenomena. Also included are articles that bridge the gap between these two perspectives, seeking synergistic links between theory and modeling and computation. The volume offers direct insight into recent trends in PDEs.

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