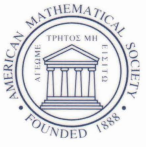
CONTEMPORARY MATHEMATICS

268

Differential Geometric Methods in the Control of Partial Differential Equations

1999 AMS-IMS-SIAM Joint Summer Research Conference
on Differential Geometric Methods in the Control
of Partial Differential Equations
University of Colorado, Boulder
June 27–July 1, 1999

Robert Gulliver
Walter Littman
Roberto Triggiani
Editors



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Providence, Rhode Island

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Preface

This volume contains selected papers which were presented at the AMS-IMS-SIAM Joint Summer Research Conference on "Differential Geometric Methods in the Control of Partial Differential Equations," which was held at the University of Colorado, Boulder, June 27–July 1, 1999.

The aim of the Conference was to explore the infusion of differential-geometric methods into the analysis of control theory of partial differential equations, particularly in the challenging case of variable coefficients, where the physical characteristics of the medium vary from point to point. While a mutually profitable link had been long established, for at least 30 years, between differential geometry and control of ordinary differential equations, a comparable relationship between differential geometry and control of partial differential equations (PDE's) is a new and promising topic. Very recent research, just prior to the Colorado Conference, supported the expectation that differential geometric methods, when brought to bear on classes of PDE modelling and control problems with variable coefficients, will yield significant mathematical advances.

Although the subject of boundary control of PDE's is about a quarter of a century old, and that of Riemannian geometry is much older still, there has been relatively little interaction between the two. It was just over 10 years ago that the role that bicharacteristics play in boundary control of hyperbolic PDE's was brought to the forefront. This then naturally leads one to think about their geometric equivalent, at least for time-invariant, second-order hyperbolic equations—geodesics, a basic concept in Riemannian geometry. Accordingly, we expect that other local and global geometrical invariants such as various notions of curvature, isoperimetric constants, uniqueness of the geodesics, etc., will be seen to be relevant. Each of these two disciplines—control theory of PDE's, and Riemannian geometry—has been pursued in virtually complete independence, or occasionally even ignorance, of the other. One of the motivations which led us to organize this Conference was the belief that both subjects have much to gain by closer interaction with one another. This, in particular, may turn out to be true in providing intrinsic (coordinate invariant) and mathematically manageable models in shell theory. On the one hand, we anticipate that the reservoir of as yet untapped Riemann-geometric methods and concepts could be applied productively in boundary control. Several of the papers in these Proceedings demonstrate such applications. On the other hand, we expect that certain problems which have arisen recently in the boundary control of PDE will stimulate the disciplines of Riemannian and Lorentzian geometry to undertake new areas of research. We will name just two examples here. First, the study of time-like Lorentzian geodesics has until now been principally motivated by cosmological problems, which, while important, are of a quite different nature

from those which arise in the control of hyperbolic PDE's with time-varying coefficients. Second, there are numerous geometric issues which arise in the optimal design of boundary control problems, few, if any, of which appear to have come to the attention of researchers in Riemannian and Lorentzian geometry.

For these and related reasons, the Conference brought together specialists and junior scientists in differential geometry, in PDE's and in boundary control theory, to join in becoming acquainted with the tools of the other disciplines, in overcoming interdisciplinary barriers, and ultimately in thrusting forward this mathematical area of research.

The papers presented at the Conference, in particular those enclosed in this volume, collectively support the claim that the aims of the Conference are being fulfilled. Each paper has a geometric flavor and component; some dig deeply into the interaction between geometric tools and the evolution of control problems. The papers feature the following topics: stabilization for structural acoustic problems and systems of anisotropic elasticity; intrinsic geometric models for shell equations; free boundary problems arising in optimization of composite membranes; evolution of a graph by its Levi form; prescribed scalar curvature on compact manifolds with boundary; sensitivity analysis; exact controllability of hyperbolic equations and related inverse problems; local and global uniqueness of PDE's (or, equivalently, approximate controllability of the adjoint system); Carleman estimates and their implications.

The organizers thus expect that the Colorado Conference will serve as a springboard for further research activities in this area and for further conferences to follow.

The conference web page, including the conference schedule, abstracts of the main talks and a participant list with email addresses, will be maintained for the next few years at:

`http://www.ima.umn.edu/ gulliver/confs/control.html`

The Conference was funded by the National Science Foundation, whose support is gratefully acknowledged.

The organizers are grateful to all participants for their contributions to the Conference, either by lecturing, by publishing in the present Proceedings, or by actively taking part in the intellectual exchange of ideas debated at the Conference.

Very warm thanks are extended to the AMS staff, and, in particular, to Mr. Wayne Drady and Ms. Donna Salter, whose much appreciated efforts and smooth, professional coordination of a large variety of activities were essential to the success of the conference.

Finally, we wish to thank Ms. Christine Thivierge from the AMS Publication Office, for precious help in connection with the publication of the present volume.

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