CONTEMPORARY MATHEMATICS

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Groups of Homotopy Self-Equivalences and Related Topics

Proceedings of the Workshop on Groups of Homotopy Self-Equivalences and Related Topics September 5–11, 1999 University of Milan, Gargnano, Italy

> Ken-ichi Maruyama John W. Rutter Editors



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This volume contains the proceedings of the Workshop on "Groups of Homotopy Self-Equivalences and Related Topics," which was held at the University of Milan, Gargnano, Italy, September 5–11, 1999. The conference was made possible by a grant from the National Institute for Advanced Mathematics "Francesco Severi" (INdAM).

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Foreword

From September 5 to 11, 1999, a workshop on groups of homotopy self-equivalences and related topics was held at the Gargnano conference centre of the University of Milan, located in the town of Gargnano (Lake Garda). The workshop was attended by some 30 mathematicians from Canada, England, Germany, Italy, Japan, Korea, the Republic of South Africa, Singapore, Slovenia, Spain and the United States. This volume contains papers presented to the workshop. It contains two specially written articles—a review of work done in the area of homotopy self-equivalences since the Montreal conference of 1988, including a bibliography, and a list of problems.

The workshop was made possible by a grant from the National Institute for Advanced Mathematics "Francesco Severi" (INdAM). Thanks are due to Martin Arkowitz and Renzo Piccinini who were main organisers before and during the workshop. Each research paper in this volume was carefully refereed and thanks are due to all who helped in this.

Ken-ichi Maruyama and John W. Rutter

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Ken-ichi Maruyama and John W. Rutter, Editors

This volume offers the proceedings from the workshop held at the Gargnano Institute of the University of Milan (Italy) on groups of homotopy self-equivalences and related topics. The book comprises articles of current research on the group of homotopy self-equivalences, the homotopy of function spaces, rational homotopy theory, the classification of homotopy types, and equivariant homotopy theory.

Mathematicians from many areas of the globe attended the workshops to discuss their research and to share ideas. Included are two specially-written articles, by J. W. Rutter, reviewing the work done in the area of homotopy self-equivalences since 1988. Included also is a bibliography of some 122 articles published since 1988 and a list of problems. This book is suitable for both advanced graduate students and researchers.

