CONTEMPORARY MATHEMATICS

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Advances in Algebraic Geometry Motivated by Physics

AMS Special Session on
Enumerative Geometry in Physics
April 1–2, 2000
University of Massachusetts, Lowell, Massachusetts

Emma Previato
Editor

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Emma Previato
Editor

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This volume contains the proceedings of the AMS Special Session on Enumerative Geometry in Physics held at The University of Massachusetts, Lowell, Massachusetts on April 1–2, 2000.

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INTRODUCTION

Emma Previato

"Every time someone invokes you, it is by a different name."
(*The Muses*, from *Dialogues with Leucò*, by C. Pavese)

When you try to tell your undergraduates what algebraic geometry is about, you probably begin with the Greeks, as does Dieudonné [D], the cissoid being a prime example of solution by geometry of an algebraic equation, the duplication of the cube. But the progress of mathematical thought keeps revealing different names of algebraic geometry, beyond what perhaps might have been the core discipline, namely Commutative Algebra: Complex Analysis, Differential Geometry, Number Theory, Representation Theory, Model Theory, Skew Rings are some of the names by which this "mother" who "existed before there were gods", like Pavese's Mnemosyne, is now invoked. And the previous list does not even include the discipline which is at the heart of this collection, Mathematical Physics.

The reciprocal influence of geometry and physics is especially crucial today: it has truly changed the face of geometry, and this change has called to life anew certain classical problems that had been shelved when the old techniques ran out of computational power. Arguably the trend began in the late '60s: Mumford liked the fact that "initial insight behind this and related discoveries was the work of the first electronic computer!" [M], which allowed a group of applied mathematicians in Princeton to simulate solutions of a non-linear wave equation, Korteweg-de Vries (KdV for short), and discover a surprising stability property: for very large time, the sum of two solutions ("solitons") was approximately a solution. Eventually this phenomenon, like Fagnano's duplication of the arc of the lemniscate [S, §I.1], was interpreted as the Abelian sum on a Jacobian: algebraic geometry all over again. Links between finite and infinite-dimensional, completely integrable Hamiltonian systems ensued; the PDE was also interpreted by M. Sato as the Plücker relations for an infinite-dimensional Grassmannian; representation theory, W-algebras, Clifford algebras and symplectic reduction progressively fell into place. N.J. Hitchin's generalization of the differential-geometric framework for the PDE gave a new thrust to the theory of moduli spaces of vector bundles; geometric quantization [H] and conformal blocks instigated the use of infinite-dimensional algebraic geometry [BL]. A deep, slowly unfolding analogy of operations on bundles with number theory was represented at the Lowell meeting (see Gaitsgory's abstract) but there was no time to include it in this book. This multiple-front development shows no sign of abating, as random matrices and Brauer classes associated to orbit problems, for example, come to the fore. In parallel, Yang-Mills theory should be mentioned; it overlaps with the KdV theory but the two have not yet been 'unified'.

A second main area of growth for geometry is connected with quantum field theory. The most robust geometric application is intersection theory on moduli spaces; new invariants (Donaldson or Gromov-Witten); new algebraic structures (Frobenius manifolds, Hopf algebras, quantum cohomology) on old spaces. But you could argue that this harks back to integrable systems, for the connections exist (KdV, Toda, Virasoro and monodromy equations) though they are still mysterious.

There is a third way, however, in which geometry was revitalized by physics; so this spring when I had the privilege of organizing a special session of the AMS meeting in Lowell, MA, I decided to learn "with a little help from my friends" (Ringo Starr) how old problems of enumerative geometry were solved by new ideas of mirror symmetry. The two Lowell days of cutting-edge talks, representing such diverse and strong-growing areas, energized us enough that we decided to put together this book. Ravi Vakil suggested the title. The timeline for proceedings is rigid, but the work came in as planned, to give both a snapshot of what is known, and a set of previously unpublished results. Each article was refereed by an expert.

Some of the Lowell participants were unable to produce an article by the end of the summer, so to give an idea of what we learned I compiled the abstracts as an Appendix, in the order they were delivered. Some of the present authors were unable to be physically present in Lowell but were so good as to contribute their written work.

In the first chapter you will find results of enumerative nature, from counting automorphisms, a contribution by Sándor Kovács, to issues of reality conditions in counting curves, a contribution by Frank Sottile, who found an alternative approach to quantum cohomology techniques in the literature of systems theory; Alexander Suciu provides a survey of numerical invariants of line arrangements in the complex plane, as well as new examples and conjectures. In chapter 2, I grouped work that has to do with deformation theory: Dan Abramovich and Aaron Bertram prove by deformation a coincidence of Gromov-Witten invariants for certain rational surfaces; Dan Abramovich and Frans Oort give results on the Hurwitz stack, parametrizing simply branched covers of the projective line, in mixed characteristic; Lucia Caporaso investigates numerical properties of tangent hyperplanes to canonical curves, in a way that is compatible with degenerations; Eduardo Cattani and Javier Fernandez establish a correspondence between quantum potentials and polarized variations of Hodge structures in a given asymptotic sense; Herbert Clemens shows by deformation theory that certain families of rational curves in projective space are stably rational; Ravi Vakil introduces certain graph invariants to reproduce results on the degeneration of a family of maps from nodal curves to a surface. Chapter 3 is loosely titled mirror symmetry and Gromov-Witten invariants, although these phenomena were already present in the previous chapters. David Cox, Sheldon Katz and Yuan-Pin Lee discuss a conjecture of Andreas Gathmann for a certain virtual fundamental class and prove it in some cases; Tyler

Jarvis, Takashi Kimura and Arkady Vaintrob introduce a new axiom in the theory of r-spin cohomological field theory and explore its consequences, thus providing a new geometric meaning for gravitational descendants; Bernd Kreußler provides results needed in the proof by A. Polishchuk and E. Zaslow of Kontsevich's homological mirror symmetry for elliptic curves; Anvar Mavlyutov gives a description of the cohomology of semiample hypersurfaces in a complete simplicial toric variety, which was previously unknown; Alexander Polishchuk and Arkady Vaintrob give an algebraic construction for a virtual fundamental class on the moduli space of curves with higher spin structures; Alexander Postnikov presents new properties of the Gromov-Witten invariants of complete flag varieties; Steven Rosenberg and Mihaela Vajiac use gauge theory techniques to give flat sections for the Dubrovin connection in quantum cohomology; Christopher Woodward's paper is a survey on related problems such as certain products in conjugacy classes of a Lie group, Witten's formula for the volumes of moduli spaces of flat connections on Riemann surfaces, loop groups, and Gromov-Witten invariants of flag manifolds.

Lastly, a word of thanks. Alas, words are inadequate (mine, at least) to portray the generosity, good cheer and hard work that went into the Lowell enterprise. The speakers scrambled out of a busy Boston life and into Lowell on DaylightSavingTime weekend; the writers donated some of their precious summer thinking time; the referees were startled (in September, first-day-of-classes time!) into reading highly technical work very quickly, and they did read it closely. And I would also like to thank warmly Mary Beth Ruskai, who delivered the invited address on Quantum Information Theory in Lowell and was so good as to suggest my name as a session organizer. The series Editor, Dr. Dennis DeTurk, was solicitously supportive of this volume even before it had much of a shape, and the AMS technical staff responded with the customary kindness and perfectionism, especially Christine Thivierge. I hope you enjoy the book!

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Appendix

Contemporary Mathematics
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APPENDIX: The Lowell Meeting

925nd AMS Meeting
University of Massachusetts Lowell, Lowell, Massachusetts
April 1-2, 2000

Special session on Enumerative Geometry in Physics, I

Eduardo Cattani (cattani@math.umass.edu), Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA 01003-4515, and **Javier Fernandez*** (jfernand@math.umass.edu), Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA 01003-4515. *Infinitesimal Variations of Hodge Structures at Infinity.* Preliminary report.

Abstract No. 952-14-132. Motivated by the study of variations of Hodge structures (VHS) around a maximally unipotent singular point, as in mirror symmetry, we introduce the notion of infinitesimal VHS at infinity. At a "finite" point, the data for an infinitesimal VHS is essentially an abelian subspace of a nilpotent subalgebra; at infinity one has the additional information of a monodromy cone. This allows for a finer classification of infinitesimal variations. We apply these ideas to the study of maximal variations where we now specify the degeneration at infinity. This leads to maximal VHS which are not of maximal dimension (as those of Carlson et al.).

Ravi D Vakil* (vakil@math.mit.edu), MIT Dept. of Mathematics, 77 Massachusetts Ave. Rm. 2-271, Cambridge, MA 02139. *Hodge integrals and Hurwitz numbers.*

Abstract No. 952-14-120. I'll sketch a proof (joint with Tom Graber) of a formula proposed by Ekedahl et al connecting Hodge integrals (intersections on the moduli space of pointed curves) with Hurwitz numbers (counting branched covers of the Riemann sphere; fundamentally combinatorial objects). As applications: (i) This shows that Hodge integrals are essentially combinatorial objects, and gives a means to compute them; it also yields nontrivial new relations among them. (ii) It also provides an ansatz to deal with Hurwitz numbers, with many applications, including a "machine" to verify predictions from physics and conjectures from combinatorics (joint with Goulden and Jackson).

Herb Clemens* (clemens@math.utah.edu), Mathematics Dept. Univ. of Utah, 155 So. 1400 East, Salt Lake City, UT 84112. *A local proof of Petri's conjecture at the general curve.*

Abstract No. 952-14-58. We give a proof of Petri's general conjecture on the unobstructedness of linear systems on a general curve, using only the local properties of the deformation space of the pair (C, L) where L is a holomorphic line bundle over a general C. We work with "Schiffer-type" variations of C, that is, deformations where the change of complex structure can be concentrated at a single point. Suppose the holomorphic sections of L are basepoint-free and that we are given a deformation L' of L over our Schiffer variation of C. Let s be a holomorphic section of L. Given a certain C-infinity trivialization $L' = L \times (t\text{-disk})$ the condition that the section (s, t) of $L \times (t\text{-disk})$ corresponds to a holomorphic section of L' becomes a sequence of conditions $D_n(s) = 0$, where D_n lies in the first cohomology group of the sheaf of holomorphic n-th order differential operators acting on sections of L. From this one concludes that, at a general curve C, the higher "mu-maps" of Arbarello-Cornalba-Griffiths-Harris all vanish. A classical argument of these same authors then gives the Petri conjecture.

Eduardo Cattani* (cattani@math.umass.edu), Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA 01003-4515, **Alicia Dickenstein** (alidick@dm.uba.ar), Departamento de Matemática, F. C. E. y N. - Universidad de Buenos Aires, Pab. I, Ciudad Universitaria, 1428 Buenos Aires, Argentina, and **Bernd Sturmfels** (bernd@math.berkeley.edu), Department of Mathematics, University of California, Berkeley, CA 94720. *Rational Hypergeometric Functions* Preliminary Report.

Abstract No. 952-33-134. Gel'fand, Kapranov and Zelevinsky have introduced a class of multivariate hypergeometric functions associated with toric varieties. We show that, generically, these varieties do not admit any rational hypergeometric functions other than Laurent polynomials. We conjecture that under appropriate assumptions all such functions are toric residues. Special cases of this conjecture will be discussed.

Special session on Enumerative Geometry in Physics, II

Tom B Graber* (graber@math.harvard.edu) and **Rahul Pandharipande**. *Quantum cohomology and quantum Chow ring of the cubic threefold.* Preliminary report.

Abstract No. 952-14-94. Although there has been a great deal of work on the Gromov-Witten theory of hypersurfaces, the cubic threefold provides an example of two phenomena that have been largely ignored. Firstly, one can calculate the Gromov-Witten invariants associated to the middle cohomology of this variety. Secondly, the cubic threefold has non-trivial Chow groups, and one can compute the quantum deformation of the ring structure on these. Both of these problems are best understood in relation to the geometry of the intermediate Jacobian.

Anvar R Mavlyutov* (anvar@math.umass.edu), Department of Mathematics, University of Massachusetts, Amherst, MA 01003. *Mirror symmetry, cohomology and the chiral ring of Calabi-Yau hypersurfaces in toric varieties.*

Abstract No. 952-14-165. One consequence of mirror symmetry is an isomorphism between the quantum cohomology on $\oplus_p H^{p,p}(X^\circ)$ and the chiral ring

$\oplus_p H^p(X, \wedge^p T_X)$ of the B-model for a pair of two distinct Calabi-Yau manifolds X and X°. We will explicitly describe a subring of the B-model chiral ring for semiample Calabi-Yau hypersurfaces X in toric varieties. This subring contains the space $H^1(X, T_X)$, which means that we have found all cocycles corresponding to deformations of smooth Calabi-Yau hypersurfaces. The chiral ring has an interesting product structure that may have a connection to the representation theory. Computation of the chiral ring goes through a description of the cohomology of the hypersurface. We will also discuss some new results in toric geometry.

Steven Rosenberg (sr@math.bu.edu), Boston University, Department of Mathematics and Statistics , 111 Cummington St., Boston, MA 02215, and **Mihaela B. Vajiac*** (mbvajiac@math.utexas.edu), University of Texas at Austin, Department of Mathematics, RLM 11.110 Mail Code C1200, Austin, TX 78712. *Gauge Theory Techniques in Quantum Cohomology I.*

Abstract No. 952-14-160. We use gauge theory to find flat sections to connections similar to the Dubrovin connection in quantum cohomology.

Steven Rosenberg* (sr@math.bu.edu), Dept. of Mathematics and Statistics, Boston University, Boston, MA 02215, and **Mihaela Vajiac** (mbvajiac@math.bu.edu), Department of Mathematics and Statistics, Boston University, Boston, MA 02215. *Gauge Theory Techniques in Quantum Cohomology II.* Preliminary report.

Abstract No. 952-14-151. We use gauge theory to find flat sections to Dubrovin connections in quantum cohomology. The techniques are analytic, involving mainly a systematic use of the Frobenius integrability theorem for matrix-valued ODEs.

Kentaro Hori* (hori@infeld.harvard.edu), Department of Physics, Harvard University, Cambridge, MA 02138. *Mirror Symmetry.*

Abstract No. 952-81-150. We derive mirror symmetry of $(2, 2)$ supersymmetric nonlinear sigma models in $1 + 1$ dimensions on toric manifolds and toric complete intersections. It involves establishing a dual description of supersymmetric abelian gauge theories. Scalar-scalar duality in $1 + 1$ dimensions and dynamical generation of superpotentials are crucial in the derivation. The mirror is generically a Landau-Ginzburg model on a non-compact Calabi-Yau manifold.

Samuel Grushevsky* (grushevs@.harvard.edu), Department of Mathematics, Harvard University, Cambridge, MA 02138. *An explicit upper bound for Weil-Petersson volumes.*

Abstract. Using λ-length coordinates introduced by R.C. Penner on the Teichmüller space of Riemann surfaces with punctures, we estimate the Weil-Petersson volume of the moduli space in the high genus limit. For the number of punctures n being fixed, and the genus g going to infinity, we show that $\mathrm{vol}(M_{g,n}) < c^g g^{2g}$, where c is an explicit constant. For the case of once-punctured surfaces our upper bound has the same upper bound as Penner's original lower bound $\mathrm{vol}(M_{g,1}) < C^g g^{2g}$, and thus gives the exact leading order growth of the Weil-Petersson volume.

Special session on Enumerative Geometry in Physics, III

Daniel Matei (dmatei@math.rochester.edu), Department of Mathematics, University of Rochester, Rochester, NY 14627, and **Alexander I Suciu***

(alexsuciu@neu.edu), Department of Matematics, Northeastern University, Boston, MA 02115. *Counting representations onto finite groups, and torsion points on characteristic varieties.*

Abstract No. 952-20-57. Given a finitely presented group G, and a finite group Γ, Philip Hall defined $\delta_\Gamma(G)$ to be the number of surjective representations from G to Γ, up to automorphisms of Γ. We show how to compute the Hall invariants of G by cohomological and combinatorial methods, when $H_1(G)$ is torsion-free, and Γ belongs to a certain class of metabelian groups. The key is provided by the stratification of the character variety, $\mathrm{Hom}(G, K^*)$, by the jumping loci of the cohomology of G, with coefficients in rank 1 local systems over a suitably chosen field K. Counting some of the torsion points on these "characteristic" varieties gives $\delta_\Gamma(G)$. In the process, we interpret the distribution of prime-index normal subgroups of G, according to their abelianization, in terms of the characteristic varieties. Under some further conditions on G, a similar count works for the subgroups of the second nilpotent quotient of G, using the "resonance" varieties associated to the cohomology ring $H^*(G; K)$. These techniques are applied to fundamental groups of complements of complex hyperplane arrangements, and to groups of singularity links of plane arrangements in $\mathbb{R}^4$.

Andras Szenes* (szenes@math.mit.edu), Massachusetts Institute of Technology, MIT 2-169, Cambridge, MA 02139. *Trigonometric Sums and Moduli Spaces.*

Abstract No. 952-05-126. We describe a residue formula for finite trigonometric sums of the Verlinde type. We apply this formula to the verification of vanishing theorems on moduli spaces of vector bundles following the work of Bismut and Labourie.

Chris T Woodward* (ctw@math.rutgers.edu), 110 Frelinghuysen Rd, Piscataway, NJ 08854, and **Constantin Teleman** (cteleman@fireant.ma.utexas.edu), Department of Mathematics RLM 8.100, Austin, TX 78712-1082. *Parabolic G-bundles and products of conjugacy classes.*

Abstract No. 952-58-90. We describe an extension of some results of Bhosle/Ramanathan, on a correspondence between parabolic G-bundles and flat bundles on a punctured surface, with fixed holonomies around the punctures. This is applied to describing which conjugacy classes appear in products of conjugacy classes for a compact connected Lie group. The answer is given in terms of quantum cohomology of generalized flag varieties.

Frank Sottile* (sottile@math.wisc.edu), Department of Mathematics, University of Wisconsin, Madison, WI 53706. *Real Rational Curves in Grassmannians.*

Abstract No. 952-14-80. In this talk, I will describe how a certain problem of enumerating rational curves on a Grassmannian (whose intersection number may be computed in the small quantum cohomology ring) may have all of its solutions be real.

Special session on Enumerative Geometry in Physics, IV

Dennis Gaitsgory* (gaitsgde@math.harvard.edu). *Geometric Langlands conjecture for $GL(n)$.* Preliminary report.

Abstract No. 952-14-161. Let X be an algebraic curve and let Bun_n be the moduli space of n-bundles on X. The geometric Langlands conjecture connects (roughly) the set of all n-dimensional representations of $\pi_1(X)$ to the category of perverse sheaves on Bun_n; the latter can be thought of as a geometric analog of the space of automorphic functions on the adelic $GL(n)$. In the talk we will recall the formulation of the conjecture and describe ideas that lead to the proof. These ideas combine purely geometric methods as well as the recent fundamental results of L. Lafforgue.

Ravi D Vakil* (vakil@math.mit.edu), MIT Dept. of Mathematics, 77 Massachusetts Ave. Rm. 2-271, Cambridge, MA 02139. *Smoothings of maps from nodal curves to nonsingular surfaces.*

Abstract No. 952-14-114. Suppose $f : C \to S$ is a morphism from a nodal curve to a nonsingular surface. We'll give a necessary (but not sufficient) numerical criterion for f to be smoothable, i.e., for f to be a special fiber in a family whose general source curve is nonsingular: roughly speaking, in such a family, the difference between the delta invariant of the special fiber and that of the general fiber is the genus of a "dual graph" of contracted components. We'll give applications, and examples of such maps that can be smoothed in many ways (i.e. whose deformation space is not unibranch).

David A. Cox* (dac@cs.amherst.edu), Dept. of Math & CS, Amherst College, P.O. Box 5000, Amherst, MA 01002-5000. *The Virtual Fundamental Class of a Complete Intersection.* Preliminary report.

Abstract No. 952-14-85. The Mirror Theorem for the quintic threefold X, as formulated by Givental and Lian, Liu and Yau, gives an explicit computation of the instanton numbers of X. A key element in the proof is a formula of Kontsevich which expresses the Gromov-Witten invariants of X as the integral of a certain Chern class over the space of stable maps on $\mathbb{P}^4$. What lurks behind this formula is an identity which computes the virtual fundamental class of stable maps on X in terms of the virtual fundamental class of stable maps on $\mathbb{P}^4$. One can also state a general version of this identity, which would apply to any complete intersection X in a smooth ambient space Y. This identity is known to hold when the ambient space is convex (e.g. $Y = \mathbb{P}^4$). Sheldon Katz and I conjecture that the identity is true in general. The talk will end with a discussion of how this identity relates to the more general mirror theorems of Givental and Lian, Liu and Yau.

Tyler Jarvis, Dept. of Mathematics, Brigham Young University, Provo, UT 84602, **Takashi Kimura*** (kimura@math.bu.edu), Dept. of Mathematics, Boston University, 111 Cummington St., Boston, MA 02215, and **Arkady Vaintrob**, Dept. of Mathematical Sciences, New Mexico University, Las Cruces, NM 88003. *The generalized Witten conjecture, integrable hierarchies, and moduli spaces of higher spin curves.*

Abstract No. 952-14-133. We precisely formulate and prove the genus zero part of the generalized Witten conjecture which states that certain intersection numbers on the moduli space of higher spin curves assemble into a generating function which solves the r-th Gelfand-Dickey (or r-th KdV) integrable hierarchies. We also

formulate axioms which insure that the correlators of the theory satisfy factorization properties analogous to those from the theory of Gromov-Witten invariants.

Alexander Polishchuk* (apolish@math.bu.edu), Department of Mathematics and Statistics, Boston University, 111 Cummington Street, Boston, MA 02215, and **Arkady Vaintrob** (vaintrob@math.nmsu.edu), Department of Mathematics, New Mexico State University, Las Cruces, NM 88003. *Algebraic construction of the virtual class for spin cohomological field theories.* Preliminary report.

Abstract No. 952-14-136. We present an algebraic construction of the virtual class on the moduli space of curves with higher spin structures. The corresponding analytic construction is due to Witten. Our main tool is excess intersection theory developed by Fulton.

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Advances in Algebraic Geometry Motivated by Physics

Emma Previato, Editor

Our knowledge of objects of algebraic geometry such as moduli of curves, (real) Schubert classes, fundamental groups of complements of hyperplane arrangements, toric varieties, and variation of Hodge structures, has been enhanced recently by ideas and constructions of quantum field theory, such as mirror symmetry, Gromov-Witten invariants, quantum cohomology, and gravitational descendants.

These are some of the themes of this refereed collection of papers, which grew out of the special session, "Enumerative Geometry in Physics," held at the AMS meeting in Lowell, MA, April 2000. This session brought together mathematicians and physicists who reported on the latest results and open questions; all the abstracts are included as an Appendix, and also included are papers by some who could not attend.

The collection provides an overview of state-of-the-art tools, links that connect classical and modern problems, and the latest knowledge available.

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