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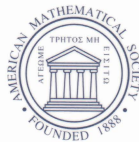
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Celestial Mechanics

Dedicated to Donald Saari
for his 60th Birthday

Proceedings of an International Conference
on Celestial Mechanics
December 15–19, 1999
Northwestern University
Evanston, Illinois

Alain Chenciner
Richard Cushman
Clark Robinson
Zhihong Jeff Xia
Editors



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American Mathematical Society
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This volume contains the proceedings of an international conference on Celestial Mechanics, which was held at Northwestern University, Evanston, Illinois on December 15–19, 1999 to celebrate Donald Saari's 60th birthday.

2000 *Mathematics Subject Classification*. Primary 70Fxx, 70Hxx, 37N05, 37Jxx.

Library of Congress Cataloging-in-Publication Data

Celestial mechanics : dedicated to Donald Saari for his 60th birthday : proceedings of an international conference on celestial mechanics, December 15–19, 1999, Northwestern University, Evanston, Illinois / Alain Chenciner . . . [et al.], editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 292)

Includes bibliographical references.

ISBN 0-8218-2902-5 (alk. paper)

1. Celestial mechanics—Congresses. I. Saari, D. (Donald) II. Chenciner, Alain. III. Contemporary mathematics (American Mathematical Society) ; v. 292.

QB349.C45 2002
521—dc21

2001055364

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Preface

From December 15 through December 19, 1999, an international conference on celestial mechanics was held at Northwestern University, Evanston, Illinois to celebrate Don Saari's upcoming 60th birthday. With admiration and appreciation over Saari's influence on the subject, and sharing a common fascination for this field, many leading experts and active researchers came to Evanston and presented their recent results. About 30 lectures were given, part of which gave rise to the written texts included in this volume.

Saari once wrote, with his unique humor, that astronomy and hence celestial mechanics form the second oldest profession of the mankind. Indeed, our fascination with motions of celestial bodies can be traced back as far as the beginning of civilizations. Newton's discovery of universal gravitation founded the modern theory of celestial mechanics and the Newtonian n -body problems. The early pioneers, such as Laplace, Lagrange, Poincaré and others put the subject into a mathematical framework, besides the obvious practical applications. The subsequent efforts of the twentieth century mathematicians like Chazy, Von Zeipel, Birkhoff, Kolmogorov, Seigel, Moser, Arnold, Alekseev and many others not only tremendously increased our understanding of the subject, but also continued to inspire the development of other fields of mathematics.

Saari's significant contribution to the field came in the late 60's, through a series of important works, some jointly with Pollard. Saari's work revived the singularity theory in the n -body problem which was started by Poincaré and Painlevé. Saari's solution of the Littlewood conjecture; Saari's classifications of expanding gravitational systems and super-hyperbolic solutions; Saari's work on singularities, collision and noncollision, and on central configurations; Saari's decompositions of configurational velocities, ..., are still much studied today and are frequently heard during the course of the conference.

The lectures cover various topics of the current research interests, from central configurations to stability of periodic orbits, from variational methods to diffusion mechanisms, from the dynamics of secular systems to the global dynamics of the solar system via frequency analysis, from Hill's problem to the low energy transfer orbits and mission design in space travel, etc. This classical field is very much alive today.

We would like to take this opportunity to thank the National Science Foundation and Northwestern University for their financial support for the conference; Mrs. Melanie Rubin and Mrs. Carmen Megrant at Northwestern for taking care of

all the practical arrangements for the conference; and Ms. Christine Thivierge at AMS for helping with the publication of this volume.

Happy birthday, Don!

Alain Chenciner, Richard Cushman, Clark Robinson, Zhihong Jeff Xia

ISBN 0-8218-2902-5



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