CONTEMPORARY MATHEMATICS

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Algebraic Number Theory and Algebraic Geometry

Papers Dedicated to A. N. Parshin on the Occasion of his Sixtieth Birthday

Sergei Vostokov
Yuri Zarhin
Editors

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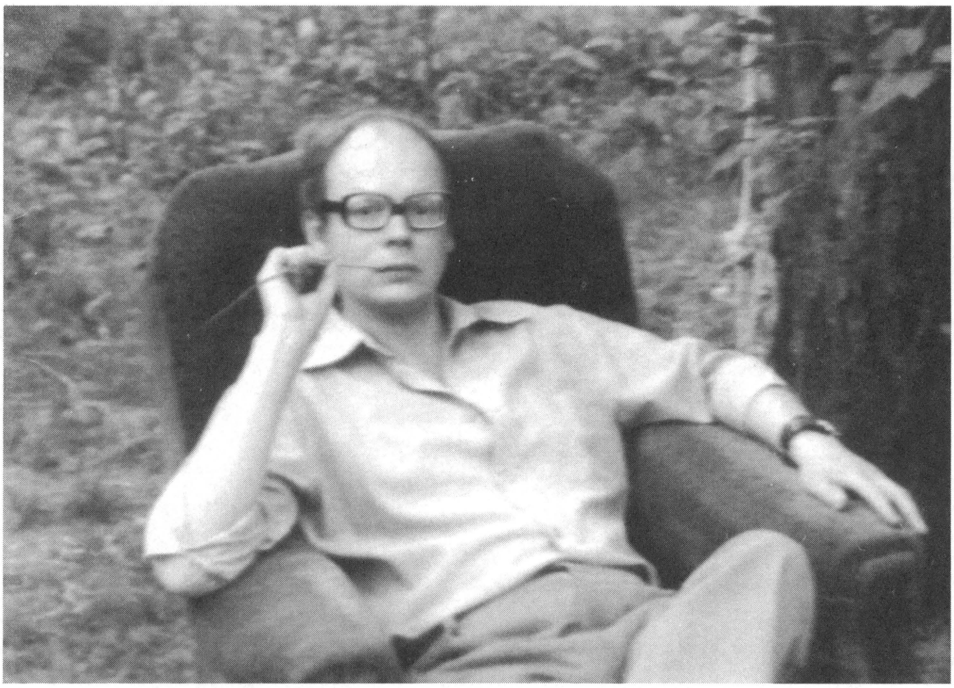

Aleksey Nikolaevich Parshin

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Contents

ALEKSEY NIKOLAEVICH PARSHIN

Aleksey Nikolaevich Parshin was born on November 7, 1942. In 1959 he entered the Mechanics and Mathematics faculty (Mekh-Mat) of Moscow State University. His outstanding mathematical capabilities became apparent already during his student years. Then he wrote a paper where he suggested a construction of a "nonabelian jacobian" of Riemann surface as the quotient of a certain (infinite-dimensional) manifold modulo the image of the fundamental group of the surface. This construction was a complex analogue of the theory of "iterated integrals" that was being developed independently by Chen. Unfortunately, A. N. Parshin never returned to this intriguing subject. Later it turned out that this theory provides impressive results for covers with nilpotent Galois groups but even in this case a lot still has to be done.

This paper was inspired by attempts to find a "nonabelian" generalization of class field theory. And all posterior papers of A. N. Parshin went back to various number-theoretical questions, though sometimes they went away to completely different fields. Mostly his research is related to the following two directions. First, it is a study of two-dimensional schemes that are "fibered" over a one-dimensional base. Parshin's attention was attracted by so called "finiteness conjecture" according to which there are only finitely many of those schemes if the base, degenerations, and the genus of the fiber are fixed (modulo obvious exceptions). His first results dealt with the case of schemes over a field, i.e., with algebraic surfaces. Parshin proved that if the conditions of the "finiteness conjecture" are fulfilled then the corresponding surfaces consist of a a finite number of algebraic families (the fact that these families are zero-dimensional, i.e. boil down to finitely many surfaces, was before long proved by S. Yu. Arakelov with help of a special deformation technique). But even Parshin's result allowed already to prove the "finiteness conjecture" for surfaces over finite fields. In his ICM talk in Nice, A. N. Parshin reported on a quite unexpected result: the validity of "finiteness conjecture" implies the celebrated "Mordell conjecture". In particular, this provides a proof of an analogue of the Mordell conjecture for schemes over finite fields. Thirteen years later Faltings proved the "finiteness conjecture" for arithmetical schemes. Thanks to the result of Parshin, this gave a proof of the Mordell conjecture. A conjectural "Parshin's inequality" (an analogue of Bogomolov-Miyaoka-Yao inequality for algebraic surfaces) lies in the same circle of ideas related to two-dimensional schemes. The validity of Parshin's inequality would imply a plentitude of arithmetical corollaries.

Another direction that A. N. Parshin continues to develop to this day is related to a generalization of class field theory and classical arithmetic of one-dimensional schemes on the whole to the higher-dimensional case. An idea of this generalization is that the basic notion is neither a point (an irreducible zero-dimensional scheme) nor an irreducible divisor (a subscheme of codimension 1) but a flag, i.e, a point, an irreducible curve passing through this point, an irreducible surface passing through this curve etc. This conception turned out to be extremely fruitful, circulating in a plenty of subjects of mathematics.

Several years were devoted by Parshin to the development of an analogue of class field theory for higher-dimensional fields. Here the main novelty was that the role of the multiplicative group of a field is played by the group K_n (more precisely, a certain topological variant of K_n defined in terms of "symbols"). This leads to a natural generalization of the classical reciprocity law. In the "geometric" case one gets new relations between residues of differential forms.

During the following years A. N. Parshin discovered and studied other interrelations between the problems described above. In particular, the building of class field theory had required the construction of an analogue of the group of adeles (or ideles) for a n-dimensional field. It turned out that this construction can be generalized to the case of adele groups of algebraic groups. In particular, the "adelic" description of vector bundles over an algebraic curve (Weil - Serre) has a precise n-dimensional analogue (for instance, for algebraic surfaces). Thus he discovered an analogue of Serre's theorem about interrelations between vector bundles and Bruhat-Tits buildings. In the same circle of ideas lies a "fixed point formula" for actions of tori on manifolds with arbitrary singularities. In number-theoretical situation these considerations allow one to study local constants in the functional equations for L-functions.

Recently many interrelations with other ideas were discovered. For example, even in the case of algebraic varieties over the field of complex numbers, the generalizations of reciprocity laws turn out to be related with so called "complex linking numbers", local symbols are related with topological conformal field theory, etc. It seems that there is a very broad area for study, whose boundary is still not visible.

Some of Parshin's papers do not fit into the two directions described above. Thus he discovered the possibility of an application of Sh. Kobayashi's hyperbolic geometry to number theoretical problems (for the first time, to the best of my knowledge). This was a beginning of now widely ramified direction: proofs of arithmetical properties of algebraic varieties (or statements of conjectures).

Parshin's interests are much broader that one may judge by his research in number theory and algebraic geometry. This is demonstrated by his participation in editing (in Russian) the collected papers of H. Weyl and D. Hilbert, his paper on complementarity principle in "Voprosy philosophii", or by his "Reflections on Gödel's Theorem" published in "Istoriko-Matematicheskie Issledovaniya".

Aleksey Nikolaevich Parshin is a corresponding member of the Russian Academy of Sciences and doctor honoris causa of the Université Paris-Nord.

I. R. Shafarevich

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A. N. Parshin is a world-renowned mathematician who has made significant contributions to number theory through the use of algebraic geometry. Articles in this volume present new research and the latest developments in algebraic number theory and algebraic geometry and are dedicated to Parshin's sixtieth birthday. Well-known mathematicians contributed to this volume, including, among others, F. Bogomolov, C. Deninger, and G. Faltings.

The book is intended for graduate students and research mathematicians interested in number theory, algebra, and algebraic geometry.

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