CONTEMPORARY MATHEMATICS

373

Analyzable Functions and Applications

International Workshop
on Analyzable Functions and Applications
June 17–21, 2002
International Centre for Mathematical Sciences
Edinburgh, Scotland

O. Costin
M. D. Kruskal
A. Macintyre
Editors

American Mathematical Society

http://dx.doi.org/10.1090/conm/373

Analyzable Functions and Applications

CONTEMPORARY MATHEMATICS

373

Analyzable Functions and Applications

International Workshop
on Analyzable Functions and Applications
June 17–21, 2002
International Centre for Mathematical Sciences
Edinburgh, Scotland

O. Costin
M. D. Kruskal
A. Macintyre
Editors

American Mathematical Society
Providence, Rhode Island

2000 *Mathematics Subject Classification.* Primary 03C64, 34M37, 34M30, 58J37, 34D15, 40G10, 39A05.

Library of Congress Cataloging-in-Publication Data

International Workshop on Analyzable Functions and Applications (2002 : Edinburgh, Scotland)

Analyzable functions and applications : International Workshop on Analyzable Functions and Applications, June 17–21, 2002, International Centre for Mathematical Sciences, Edinburgh, Scotland / O. Costin, M. D. Kruskal, A. Macintyre, editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 373)

Includes bibliographical references.

ISBN 0-8218-3419-3 (alk. paper)

1. Asymptotic expansions—Congresses. 2. Functions—Congresses. I. Costin, O. (Ovidiu), 1960– II. Kruskal, Martin D. (Martin David), 1925– III. Macintyre, A. (Angus), 1941– IV. Title. V. Contemporary mathematics (American Mathematical Society) ; v. 373.

QA355.I58 2002
515—dc22 2004062790

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 10 09 08 07 06 05

Contents

Preface

Asymptotic analysis is extensively used to explore systems for which exact solutions are not available or are difficult to manipulate. In most cases of interest, however, classical asymptotics does not provide global information; in particular it does not yield information at values of the parameters which are not large, small, or otherwise special. The information provided by classical expansions is not complete; every classical asymptotic series represents a large class of functions. Furthermore, classical expansions are not closed under all common operations. Larger structures are needed to represent faithfully functions occurring in applications.

Transseries and LE series, two very related structures, are formally asymptotic expansions of ordinal length of power series, exponentials, and logs. They form a field closed under most usual operations, including algebraic ones, functional ones like composition and functional inversion, and analytical ones including differentiation and integration. As a consequence, they have the natural richness to represent a large class of functions including quite general solutions of differential and difference equations and some types of partial differential equations. In the theory of analyzable functions, transseries are precisely defined, studied, and can be relatively generally "summed" to actual functions. Transseries contain global information about the functions they represent.

Key ideas in the direction of a theory of analyzable functions are present in the works of Euler, Cauchy, Stokes, Hardy, Borel, and others. The theory took a great leap forward in the early 1980s with the work of J. Écalle; similar techniques and concepts emerged at essentially the same time in analysis, logic, applied mathematics and surreal number theory and developed rapidly through the 1990s.

The links between the various approaches soon became apparent and this body of ideas is now recognized as a field of its own with numerous nontrivial applications. The contributions to this volume are a sequel of the International Workshop on Analyzable Functions and Applications, Edinburgh, June 17–21, 2002, in which experts from many areas of mathematics participated.

The meeting was possible through generous funding by UK's Engineering and Physical Sciences Research Council.

We thank the International Centre for Mathematical Sciences and in particular Mrs. Tracey Dart for their great help and cooperation.

The Authors

Titles in This Series

The theory of analyzable functions is a technique used to study a wide class of asymptotic expansion methods and their applications in analysis, difference and differential equations, partial differential equations and other areas of mathematics.

Key ideas in the theory of analyzable functions were laid out by Euler, Cauchy, Stokes, Hardy, E. Borel, and others. Then in the early 1980s, this theory took a great leap forward with the work of J. Écalle. Similar techniques and concepts in analysis, logic, applied mathematics and surreal number theory emerged at essentially the same time and developed rapidly through the 1990s. The links among various approaches soon became apparent and this body of ideas is now recognized as a field of its own with numerous applications.

This volume stemmed from the International Workshop on Analyzable Functions and Applications held in Edinburgh (Scotland). The contributed articles, written by many leading experts, are suitable for graduate students and researchers interested in asymptotic methods.

ISBN 0-8218-3419-3

9 780821 834190

CONM/373