

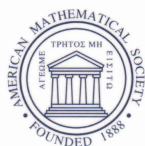
CONTEMPORARY MATHEMATICS

373

Analyzable Functions and Applications

International Workshop
on Analyzable Functions and Applications
June 17–21, 2002
International Centre for Mathematical Sciences
Edinburgh, Scotland

O. Costin
M. D. Kruskal
A. Macintyre
Editors



Analyzable Functions and Applications

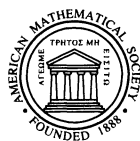
CONTEMPORARY MATHEMATICS

373

Analyzable Functions and Applications

International Workshop
on Analyzable Functions and Applications
June 17–21, 2002
International Centre for Mathematical Sciences
Edinburgh, Scotland

O. Costin
M. D. Kruskal
A. Macintyre
Editors



American Mathematical Society
Providence, Rhode Island

Editorial Board

Dennis DeTurck, managing editor

Andreas Blass Andy R. Magid Michael Vogelius

2000 *Mathematics Subject Classification*. Primary 03C64, 34M37, 34M30, 58J37, 34D15, 40G10, 39A05.

Library of Congress Cataloging-in-Publication Data

International Workshop on Analyzable Functions and Applications (2002 : Edinburgh, Scotland)

Analyzable functions and applications : International Workshop on Analyzable Functions and Applications, June 17–21, 2002, International Centre for Mathematical Sciences, Edinburgh, Scotland / O. Costin, M. D. Kruskal, A. Macintyre, editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 373)

Includes bibliographical references.

ISBN 0-8218-3419-3 (alk. paper)

1. Asymptotic expansions—Congresses. 2. Functions—Congresses. I. Costin, O. (Ovidiu), 1960– II. Kruskal, Martin D. (Martin David), 1925– III. Macintyre, A. (Angus), 1941– IV. Title. V. Contemporary mathematics (American Mathematical Society) ; v. 373.

QA355.I58 2002
515—dc22

2004062790

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to reprint-permission@ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2005 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.

Copyright of individual articles may revert to the public domain 28 years
after publication. Contact the AMS for copyright status of individual articles.

Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 10 09 08 07 06 05

Contents

Preface	vii
A singularly perturbed Riccati equation SADJIA AÏT-MOKHTAR	1
On global aspects of exact WKB analysis of operators admitting infinitely many phases TAKASHI AOKI, TAKAHIRO KAWAI, TATSUYA KOIKE, AND YOSHITSUGU TAKEI	11
Asymptotic differential algebra MATTHIAS ASCHENBRENNER AND LOU VAN DEN DRIES	49
Formally well-posed Cauchy problems for linear partial differential equations with constant coefficients WERNER BALSER AND VLADIMIR KOSTOV	87
Non-oscillating integral curves and o-minimal structures F. BLAIS, R. MOUSSU, AND J.-P., ROLIN	103
Asymptotics and singularities for a class of difference equations BOELE BRAAKSMA AND ROBERT KUIK	113
Topological construction of transseries and introduction to generalized Borel summability O. COSTIN	137
Addendum to the hyperasymptotics for multidimensional Laplace integrals E. DELABAERE	177
Higher-order terms for the de Moivre-Laplace theorem FRANCINE DIENER AND MARC DIENER	191
Twisted resurgence monomials and canonical-spherical synthesis of local objects JEAN ECALLE	207
Matching and singularities of canard values A. FRUCHARD AND E. MATZINGER	317
On the renormalization method of Chen, Goldenfeld, and Oono BLESSING MUDAVANHU AND ROBERT E. O'MALLEY, JR.	337

Generalizing surreal numbers S. P. NORTON	347
Two examples of resurgence C. OLIVÉ, D. SAUZIN, AND T. M. SEARA	355

Preface

Asymptotic analysis is extensively used to explore systems for which exact solutions are not available or are difficult to manipulate. In most cases of interest, however, classical asymptotics does not provide global information; in particular it does not yield information at values of the parameters which are not large, small, or otherwise special. The information provided by classical expansions is not complete; every classical asymptotic series represents a large class of functions. Furthermore, classical expansions are not closed under all common operations. Larger structures are needed to represent faithfully functions occurring in applications.

Transseries and LE series, two very related structures, are formally asymptotic expansions of ordinal length of power series, exponentials, and logs. They form a field closed under most usual operations, including algebraic ones, functional ones like composition and functional inversion, and analytical ones including differentiation and integration. As a consequence, they have the natural richness to represent a large class of functions including quite general solutions of differential and difference equations and some types of partial differential equations. In the theory of analyzable functions, transseries are precisely defined, studied, and can be relatively generally “summed” to actual functions. Transseries contain global information about the functions they represent.

Key ideas in the direction of a theory of analyzable functions are present in the works of Euler, Cauchy, Stokes, Hardy, Borel, and others. The theory took a great leap forward in the early 1980s with the work of J. Écalle; similar techniques and concepts emerged at essentially the same time in analysis, logic, applied mathematics and surreal number theory and developed rapidly through the 1990s.

The links between the various approaches soon became apparent and this body of ideas is now recognized as a field of its own with numerous nontrivial applications. The contributions to this volume are a sequel of the International Workshop on Analyzable Functions and Applications, Edinburgh, June 17–21, 2002, in which experts from many areas of mathematics participated.

The meeting was possible through generous funding by UK’s Engineering and Physical Sciences Research Council.

We thank the International Centre for Mathematical Sciences and in particular Mrs. Tracey Dart for their great help and cooperation.

The Authors

Titles in This Series

- 373 **O. Costin, M. D. Kruskal, and A. Macintyre, Editors**, Analyzable functions and applications, 2005
- 372 **José Burillo, Sean Cleary, Murray Elder, Jennifer Taback, and Enric Ventura, Editors**, Geometric methods in group theory, 2005
- 371 **Gui-Qiang Chen, George Gasper, and Joseph Jerome, Editors**, Nonlinear partial differential equations and related analysis, 2005
- 370 **Pietro Poggi-Corradini, Editor**, The p -harmonic equation and recent advances in analysis, 2005
- 369 **Jaime Gutierrez, Vladimir Shpilrain, and Jie-Tai Yu, Editors**, Affine algebraic geometry, 2005
- 368 **Sagun Chanillo, Paulo D. Cordaro, Nicholas Hanges, Jorge Hounie, and Abdelhamid Meziani, Editors**, Geometric analysis of PDE and several complex variables, 2005
- 367 **Shu-Cheng Chang, Bennett Chow, Sun-Chin Chu, and Chang-Shou Lin, Editors**, Geometric evolution equations, 2005
- 366 **Bernhelm Booß-Bavnbek, Gerd Grubb, and Krzysztof P. Wojciechowski, Editors**, Spectral geometry of manifolds with boundary and decomposition of manifolds, 2005
- 365 **Robert S. Doran and Richard V. Kadison, Editors**, Operator algebras, quantization, and non-commutative geometry, 2004
- 364 **Mark Agranovsky, Lavi Karp, David Shoikhet, and Lawrence Zalcman, Editors**, Complex analysis and dynamical systems, 2004
- 363 **Anthony To-Ming Lau and Volker Runde, Editors**, Banach algebras and their applications, 2004
- 362 **Carlos Concha, Raul Manasevich, Gunther Uhlmann, and Michael S. Vogelius, Editors**, Partial differential equations and inverse problems, 2004
- 361 **Ali Enayat and Roman Kossak, Editors**, Nonstandard models of arithmetic and set theory, 2004
- 360 **Alexei G. Myasnikov and Vladimir Shpilrain, Editors**, Group theory, statistics, and cryptography, 2004
- 359 **S. Dostoglou and P. Ehrlich, Editors**, Advances in differential geometry and general relativity, 2004
- 358 **David Burns, Christian Popescu, Jonathan Sands, and David Solomon, Editors**, Stark's Conjectures: Recent work and new directions, 2004
- 357 **John Neuberger, Editor**, Variational methods: open problems, recent progress, and numerical algorithms, 2004
- 356 **Idris Assani, Editor**, Chapel Hill ergodic theory workshops, 2004
- 355 **William Abikoff and Andrew Haas, Editors**, In the tradition of Ahlfors and Bers, III, 2004
- 354 **Terence Gaffney and Maria Aparecida Soares Ruas, Editors**, Real and complex singularities, 2004
- 353 **M. C. Carvalho and J. F. Rodrigues, Editors**, Recent advances in the theory and applications of mass transport, 2004
- 352 **Marek Kubale, Editor**, Graph colorings, 2004
- 351 **George Yin and Qing Zhang, Editors**, Mathematics of finance, 2004
- 350 **Abbas Bahri, Sergiu Klainerman, and Michael Vogelius, Editors**, Noncompact problems at the intersection of geometry, analysis, and topology, 2004
- 349 **Alexandre V. Borovik and Alexei G. Myasnikov, Editors**, Computational and experimental group theory, 2004
- 348 **Hiroshi Isozaki, Editor**, Inverse problems and spectral theory, 2004

TITLES IN THIS SERIES

- 347 **Motoko Kotani, Tomoyuki Shirai, and Toshikazu Sunada, Editors**, Discrete geometric analysis, 2004
- 346 **Paul Goerss and Stewart Priddy, Editors**, Homotopy theory: Relations with algebraic geometry, group cohomology, and algebraic K -theory, 2004
- 345 **Christopher Heil, Palle E. T. Jorgensen, and David R. Larson, Editors**, Wavelets, frames and operator theory, 2004
- 344 **Ricardo Baeza, John S. Hsia, Bill Jacob, and Alexander Prestel, Editors**, Algebraic and arithmetic theory of quadratic forms, 2004
- 343 **N. Sthanumoorthy and Kailash C. Misra, Editors**, Kac-Moody Lie algebras and related topics, 2004
- 342 **János Pach, Editor**, Towards a theory of geometric graphs, 2004
- 341 **Hugo Arizmendi, Carlos Bosch, and Lourdes Palacios, Editors**, Topological algebras and their applications, 2004
- 340 **Rafael del Río and Carlos Villegas-Blas, Editors**, Spectral theory of Schrödinger operators, 2004
- 339 **Peter Kuchment, Editor**, Waves in periodic and random media, 2003
- 338 **Pascal Auscher, Thierry Coulhon, and Alexander Grigor'yan, Editors**, Heat kernels and analysis on manifolds, graphs, and metric spaces, 2003
- 337 **Krishan L. Duggal and Ramesh Sharma, Editors**, Recent advances in Riemannian and Lorentzian geometries, 2003
- 336 **José González-Barrios, Jorge A. León, and Ana Meda, Editors**, Stochastic models, 2003
- 335 **Geoffrey L. Price, B. Mitchell Baker, Palle E.T. Jorgensen, and Paul S. Muhly, Editors**, Advances in quantum dynamics, 2003
- 334 **Ron Goldman and Rimvydas Krasauskas, Editors**, Topics in algebraic geometry and geometric modeling, 2003
- 333 **Giovanni Alessandrini and Gunther Uhlmann, Editors**, Inverse problems: Theory and applications, 2003
- 332 **John Bland, Kang-Tae Kim, and Steven G. Krantz, Editors**, Explorations in complex and Riemannian geometry, 2003
- 331 **Luchezar L. Avramov, Marc Chardin, Marcel Morales, and Claudia Polini, Editors**, Commutative algebra: Interactions with algebraic geometry, 2003
- 330 **S. Y. Cheng, C.-W. Shu, and T. Tang, Editors**, Recent advances in scientific computing and partial differential equations, 2003
- 329 **Zhangxin Chen, Roland Glowinski, and Kaitai Li, Editors**, Current trends in scientific computing, 2003
- 328 **Krzysztof Jarosz, Editor**, Function spaces, 2003
- 327 **Yulia Karpeshina, Günter Stolz, Rudi Weikard, and Yanni Zeng, Editors**, Advances in differential equations and mathematical physics, 2003
- 326 **Kenneth D. T-R McLaughlin and Xin Zhou, Editors**, Recent developments in integrable systems and Riemann-Hilbert problems, 2003
- 325 **Seok-Jin Kang and Kyu-Hwan Lee, Editors**, Combinatorial and geometric representation theory, 2003
- 324 **Caroline Grant Melles, Jean-Paul Brasselet, Gary Kennedy, Kristin Lauter, and Lee McEwan, Editors**, Topics in algebraic and noncommutative geometry, 2003

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/.

The theory of analyzable functions is a technique used to study a wide class of asymptotic expansion methods and their applications in analysis, difference and differential equations, partial differential equations and other areas of mathematics.

Key ideas in the theory of analyzable functions were laid out by Euler, Cauchy, Stokes, Hardy, E. Borel, and others. Then in the early 1980s, this theory took a great leap forward with the work of J. Écalle. Similar techniques and concepts in analysis, logic, applied mathematics and surreal number theory emerged at essentially the same time and developed rapidly through the 1990s. The links among various approaches soon became apparent and this body of ideas is now recognized as a field of its own with numerous applications.

This volume stemmed from the International Workshop on Analyzable Functions and Applications held in Edinburgh (Scotland). The contributed articles, written by many leading experts, are suitable for graduate students and researchers interested in asymptotic methods.

ISBN 0-8218-3419-3



9 780821 834190

CONM/373

AMS on the Web
www.ams.org