

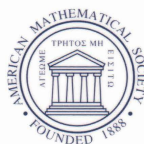
# CONTEMPORARY MATHEMATICS

384

## Ultrametric Functional Analysis

Eighth International Conference  
on  $p$ -adic Functional Analysis  
July 5–9, 2004  
Université Blaise Pascal,  
Clermont-Ferrand, France

B. Diarra  
A. Escassut  
A. K. Katsaras  
L. Narici  
Editors



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This volume contains research articles based on some of the talks given at the Eighth International Conference on  $p$ -adic Functional Analysis held at the Université Blaise Pascal of Clermont-Ferrand, France, on July 5–9, 2004.

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## Contents

Preface	vii
Vector measures and integral operators in the non-Archimedean setting JOSÉ AGUAYO	1
On the dual space for the strict topology $\beta_1$ and the space $M(X)$ in function space J. AGUAYO, A. K. KATSARAS, AND S. NAVARRO	15
Perturbation of a $p$ -adic dynamical system in two variables J. AGUAYO, J. GOMÉZ, M. SAAVEDRA, AND M. WALLACE	39
Isomorphisms with small bound between spaces of $p$ -adic continuous functions II JESÚS ARAUJO	53
Ultrametric $q$ -calculus BERTIN DIARRA	63
Strictness and closedness in $p$ -adic inductive limits N. DE GRANDE-DE KIMPE AND C. PEREZ-GARCIA	79
A note on Browkin-Brzeziński conjecture PEI-CHU HU AND CHUNG-CHUN YANG	101
Non-Archimedean integration and strict topologies A. K. KATSARAS	111
Non-Archimedean orthomodular spaces and their residual spaces HANS A. KELLER AND H. OCHSENIUS A.	145
Polylogarithms and a zeta function for finite places of a function field ANATOLY N. KOCHUBEI	157
On finite-dimensional normed spaces over $C_p$ ALBERT KUBZDELA	169
A non-Archimedean inner product LAWRENCE NARICI AND EDWARD BECKENSTEIN	187
Lipschitz operators on Banach spaces over Krull valued fields H. OCHSENIUS AND W. H. SCHIKHOF	203

Remarks on some theorems of functional analysis SIBYLLA PRIESS-CRAMPE	235
Frobenius structure for rank one $p$ -adic differential equations ANDREA PULITA	247
The ultrametric spectrum as an ordered set ALAIN SALINIER	259
Analytic roots of rational functions whose poles are on the unit circle MARIE-CLAUDE SARMANT	271
$p$ -adic Choquet Theory W. H. SCHIKHOF	281
The spherical completion of normed vector spaces over fields with valuations of arbitrary rank ERWIN SCHÖRNER	299
On Köthe quotients of non-Archimedean Fréchet spaces WIESŁAW ŚLIWA	309
Unique range sets for non-Archimedean entire functions in positive characteristic fields TA THI HOAI AN AND JULIE TZU-YUEH WANG	323
Some continuous linear operators and orthogonal $q$ -bases on the space of $p$ -adic continuous functions defined on $\mathbb{Z}_p$ FANA TANGARA	335
Uniqueness polynomials, unique range sets and other uniqueness theorems JULIE TZU-YUEH WANG	353

## Preface

The Eighth International Conference on  $p$ -adic Functional Analysis was held at the Université Blaise Pascal of Clermont-Ferrand, France, July 5–9, 2004. The first conference of this series was organized in 1990 by the University of Santander in Laredo, Spain and the series continued with meetings every two years in various places.

Now, nearly 40 researchers from 16 different countries attended the July conference. This book contains research articles based on some of the 30-minute talks given at the conference.

Basic functional analysis is represented by contributions covering a wide range of topics: Hilbert and Banach spaces, finite dimensional spaces, locally convex spaces, Choquet Theory, Fréchet spaces, inner products, orthogonality, strict topologies, spaces of continuous functions and of strictly differentiable functions, operator theory: isomorphisms between Banach functions spaces, umbral calculus and orthogonal bases, measure and integration: vector measures, dual space for strict topology, Banach and topological algebras: spectrum. Analytic and meromorphic functions: unique range sets, uniqueness polynomial, Browkin-Brzezinski conjecture, differential equations (roots of rational functions and Frobenius structure), dynamical systems, are the subject of several papers. In addition the reader will find studies on analysis in the case where the ground field is of positive characteristic: special functions, unique range sets and meromorphic functions and also studies in the case where the ground field is a Krull valued field: functional analysis, orthomodular spaces, spherical completion of normed spaces, Lipschitz operators.

We are grateful to the Société Mathématique de France for having awarded us its patronage and to the Laboratoire de Mathématiques, the UFR-RST of Université Blaise Pascal, the Conseil Régional de la Région Auvergne, the Conseil Général du Puy de Dôme, the community of townships Clermont-Communauté and to the township of Clermont-Ferrand for their financial support. We also wish to thank the American Mathematical Society for publishing this volume.

B. Diarra, A. Escassut, A.K. Katsaras, L. Narici

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With contributions by leading mathematicians, this proceedings volume reflects the program of the Eighth International Conference on  $p$ -adic Functional Analysis held at Blaise Pascal University (Clermont-Ferrand, France).

Articles in the book offer a comprehensive overview of research in the area. A wide range of topics are covered, including basic ultrametric functional analysis, topological vector spaces, measure and integration, Choquet theory, Banach and topological algebras, analytic functions (in particular, in connection with algebraic geometry), roots of rational functions and Frobenius structure in  $p$ -adic differential equations, and  $q$ -ultrametric calculus.

The material is suitable for graduate students and researchers interested in number theory, functional analysis, and algebra.

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