CONTEMPORARY MATHEMATICS

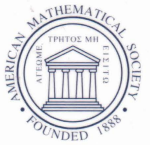
411

Harmonic Analysis

Calderón-Zygmund and Beyond

A Conference in Honor of Stephen Vági's Retirement
December 6–8, 2002
DePaul University, Chicago, Illinois

J. Marshall Ash
Roger L. Jones
Editors



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Contents

Preface

In December of 2002, a conference with the same title as this volume was held at DePaul University. Twenty-eight years earlier, in June of 1974, a similar, but more expository conference, "A survey of harmonic analysis," was held at DePaul University. A comparison of the two conferences gives an insight into how the subject has matured. In short, the talks in the earlier conference were about harmonic analysis itself, whereas the talks in the second conference are a mixture of similar talks together with talks about the application of harmonic analysis to other areas of mathematics: predominantly partial differential equations and ergodic theory.

The formal part of the conference consisted of eight one-hour talks and a problem session. Most of the articles in this book are developed versions of those nine events. The idea of the conference was to honor Stephen Vági, who had just retired from a long and distinguished career at DePaul. For this reason, two of the articles, those of Adam Korányi and Eduardo Gatto, are directly concerned with his work.

Stephen Vági is an example of a mathematician whose importance reaches far beyond his published record. Throughout his stay at DePaul, which began in the fall of 1962 and ended in the fall of 2001, he was a tremendous resource and inspiration for both students and colleagues. If one of us had a mathematical question, an excellent first approach was to ask Steve. His scholarship was remarkably broad as well as deep. He had many unpublished results and could have been coauthor of many, many works.

Another talent of Vági, not particularly mathematical, was his knowledge of foreign languages. He frequently helped with German, Russian, and several Romance languages as well. In fact, his perfect command of English is quite amazing since Hungarian, German, and Spanish are his first languages. One amusing thing that arose out of this was Vági's creation of the fictitious mathematician Donald Caccia. This was the name under which a group of DePaul mathematicians submitted solutions to a variety of MAA Monthly problems. Caccia is an Italian name that, when pronounced in Italian, sounds like the Hungarian word for duck. Perhaps all these language skills may have something to do with his very decorous personal behavior. When greeting a mathematician he hasn't seen for a while, Steve is much more likely to ask about that person's work than to begin discussing his own. For some people this is only the case when they don't have much to say, but that is definitely not the case here: Adam Korányi discusses mostly the published work of Steve Vági. This is entirely appropriate since Korányi was his first coauthor; they have eight papers in common. This is a very complete overview, beginning with his thesis and giving discussion and perspective of all of his work through the most recent eight papers, all of which were coauthored with Eduardo Gatto. There are

two bibliographies. The first lists the 23 works that Vági has published to date. This is a very nice body of work spread over 31 years and 430 pages.

When we asked Gatto (who could not attend the conference due to being away from DePaul on a visit to Spain at the time) to write an article on Vági's later work with him, he pointed out that this had already been placed in perspective very nicely by Korányi. So he instead sheds light on the Vági-Gatto program that developed a fractional calculus for spaces of homogeneous type by looking at how this program works in more general spaces. In particular, he gives some extensions of these results to the recently open area of research of measure metric spaces associated to non-doubling measures.

Charles Fefferman's talk was a large overview delivered in an informal way with great clarity. What we present here is essentially a transcript of that talk. Unfortunately, most of the great mathematicians of our day are too busy doing mathematics to give the time required for insightful exposition, so it is a pleasure to be able to share the delights of this talk with our readers. The subject is Euler's equation for the force acting on a molecule in a fluid. Fefferman has also written about this in connection with one of the Clay Mathematics Institute's seven one-million-dollar prizes. That prize is for solving the Navier-Stokes equation, a special case of which is Euler's equation. That discussion can be viewed at `http://www.claymath.org/millennium/Navier-Stokes_Equations/Official_Problem_Description.pdf`.

Carlos Kenig has been at the center of a very large project with the goal of increasing understanding of the Cauchy problem of partial differential equation for many years. Here he describes recent progress of both local and global nature in the areas of existence, uniqueness, and well-posedness. Three very important examples of the Cauchy problem are the Korteweg-de Vries equation, the Kadomstev-Petviashvili-II equations, and the nonlinear Schrodinger equation. The solving of these equations when the initial conditions are extremely rough has been one of the most successful applications of harmonic analysis.

Cora Sadosky describes a scale of BMO (bounded mean oscillation) spaces that appears naturally in product spaces, corresponding to the different, yet equivalent, characterizations of the class of functions of bounded mean oscillation in one variable. These different BMO spaces provide solutions to outstanding problems in harmonic analysis on product spaces and in multidimensional operator theory. The characterization of the endspaces in the scale, in terms of nested commutators, has provided the clue to check which functions are in product BMO. The famous 1974 Carleson counterexample, which played a significant role then in distinguishing the BMO spaces in the bidisk, continues to do so for both scalar-valued and operator-valued BMO.

Benjamin Muckenhoupt's talk is most appropriate for this conference since it describes the significant but still partially unpublished work of David Webb; and David was the last of the numerous harmonic analysts that came to work at DePaul during Vági's time there. To study the norm convergence of a Laguerre series of a function defined on the half line $(0, \infty)$, it is quite useful to have good estimates for the associated Cesaro kernels. The estimates were not found by the people who worked in this area earlier. After Webb's work, we now know that they were very hard to find because they involve a large number of cases. The most astonishing thing about this situation is that these seemingly overly complicated estimates actually are sharp and not subject to improvement in a certain well-defined sense.

Roger Jones looks into square function inequalities and variational inequalities for the Hilbert transform as well as more general singular integrals. To achieve this, he first establishes a number of properties of the variational operator, and proves results related to it. Variational inequalities give quantitative information on the convergence of truncated integrals to the corresponding transform. In particular, he obtains λ jump inequalities for singular integrals, giving estimates for the number of times the family of operators changes by more than λ as it approaches its limit. Many of these results had previously been obtained for related operators in the context of ergodic theory.

Stephen Wainger discusses work that he did jointly with Akos Magyar and Elias Stein. They first looked at the spherical maximal function of a function defined on the lattice points of d-dimensional space and ask whether the mapping from the function to its maximal function is bounded on $\ell^p(\mathbb{Z}^d)$. They then study a similar question for the d-dimensional discrete Heisenberg group. Their results have applications in ergodic theory; so Wainger also gives a very quick and light survey of some interesting facts from ergodic theory in order to give a context for those applications.

At the end of the last session of the conference, there was a problem session. Several people posed open questions and made comments on the mathematics that had been presented. We have collected some of those questions to form the last chapter of this book. Included are questions presented by A. Korányi, B. Muckenhoupt, R. Jones, P. Janakiraman, D. Ryabogin and J. Xiao.

One very fine talk given at the conference is the only one not included in this book: Elias M. Stein's "Singular integrals – Calderón-Zygmund and beyond." The reason for not including it here is that a great deal of the research program that it described as being underway has been either completed and/or has led to a deeper understanding during the three-year gap between the conference and the time that this book is going to press.

Stein pointed out that a group of mathematicians including D. Müller, A. Nagel, F. Ricci, S. Wainger, [and himself] was obtaining insights into how the classical Calderón-Zygmund singular integral theory should be extended. His main theme was that two of the main ingredients necessary for this extension were product domains and curvature. We look forward to the appearance of an updated version of this talk.

We would like to thank the National Science Foundation for grant number 0229157, which provided much of the financial support that made this conference possible. We are also indebted to DePaul University's University Research Council for substantial financial assistance and to DePaul University for the use of their facilities. Chris Novak did an excellent job of transcribing and sometimes translating in order to move some of the presentations from video to paper. Barbara Beeton of the American Mathematical Society provided a lot of technical help and encouragement.

Marshall Ash, Roger Jones
November 2005

Index

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414 **Deguang Han, Palle E. T. Jorgensen, and David Royal Larson, Editors,** Operator theory, operator algebras, and applications, 2006

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Starting in the early 1950's, Alberto Calderón, Antoni Zygmund and their students developed a program in harmonic analysis with far reaching consequences. The title of these proceedings reflects this broad reach. This book came out of a DePaul University conference honoring Stephen Vági upon his retirement in 2002. Vági was a student of Calderón in the 1960's, when Calderón and Zygmund were at their peak.

Two authors, Kenig and Gatto, were students of Calderón; one, Muckenhoupt, was a student of Zygmund. Two others studied under Zygmund's student Elias Stein. The remaining authors all have close connections with the Calderón–Zygmund school of analysis.

This book should interest specialists in harmonic analysis and those curious to see it applied to partial differential equations and ergodic theory.

In the first article, Adam Koranyi summarizes Vági's work. Four further articles cover various recent developments in harmonic analysis: Eduardo Gatto studies spaces with doubling and non-doubling measures; Cora Sadosky, product spaces; Benjamin Muckenhoupt, Laguerre expansions; and Roger Jones, singular integrals. Charles Fefferman and Carlos Kenig present applications to partial differential equations and Stephen Wainger gives an application to ergodic theory. The final article records some interesting open questions from a problem session that concluded the conference.

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CONM/411