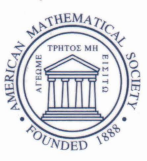
CONTEMPORARY MATHEMATICS

413

Representations of Algebraic Groups, Quantum Groups, and Lie Algebras

AMS-IMS-SIAM Joint Summer Research Conference
July 11–15, 2004
Snowbird Resort, Snowbird, Utah

Georgia Benkart
Jens C. Jantzen
Zongzhu Lin
Daniel K. Nakano
Brian J. Parshall
Editors



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Conference Group Photo
Snowbird Resort
July 2004

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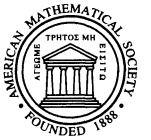

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Preface

Representation theory has played a central role in mathematics through its rich interplay with, and applications to, many other fields. The 2004 **AMS-IMS-SIAM Joint Summer Research Conference**, *Representations of Algebraic Groups, Quantum Groups, and Lie Algebras*, focused on the geometric and combinatorial aspects of the subject. New developments involving quiver representations were presented in connection with important constructions for quantum groups. Another major theme was use of methods from algebraic geometry, via derived categories, to study the representation theory of algebraic groups and Lie algebras, including Kac-Moody Lie algebras, modular restricted Lie algebras (or, more generally, finite group schemes), and Lie superalgebras.

Each morning session featured principal speakers on the designated major themes. Each afternoon, two parallel sessions allowed attendees to present talks on current research, providing a forum for junior mathematicians to communicate new developments in the area, followed by ample time for informal discussions and interaction.

The present volume brings together papers from the principal speakers and other participants on a wide variety of topics in modern representation theory. Several contributions are surveys that aim to introduce the topics to a wider audience of researchers. All of the papers were carefully refereed, and the editors express their gratitude to the anonymous referees for the high standards employed in preparing their reports.

During the conference, a banquet was held to celebrate the achievements of James E. Humphreys on the occasion of his 65th birthday. Over the last 40 years, Jim's contributions have inspired many deep insights and new developments in the representations of algebraic groups and finite groups of Lie type. In addition, his well-known books in the area have brought vast, intertwined research topics together in a concise and coherent manner. Jim has also encouraged many of us by taking a genuine interest in our work. Several months before the conference, he formally retired from the University of Massachusetts, Amherst, to devote himself to research and to book writing. We are delighted to include here one of his recent articles, which poses an interesting conjecture relating irreducible representations of semisimple Lie algebras in positive characteristics to left cells in affine Weyl groups.

Financial support for the conference was provided by a grant from the National Science Foundation, and the staff of the American Mathematical Society provided considerable logistical support. In particular, the organizers acknowledge Wayne Drady for his professional dedication to managing the conference and Christine M. Thivierge for her patience and help in editing this volume. We also thank the participants for making the conference a success: the speakers during the conference

and at the banquet and the afternoon session chairs for their work in keeping the conference on schedule. Special thanks go to Leonard Scott, whose toastmastery during the banquet provided many humorous and wonderful memories.

Georgia Benkart
Jens C. Jantzen
Zongzhu Lin
Daniel K. Nakano
Brian J. Parshall
January 2006

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This book contains several well-written, accessible survey papers in many interrelated areas of current research. These areas cover various aspects of the representation theory of Lie algebras, finite groups of Lie types, Hecke algebras, and Lie superalgebras. Geometric methods have been instrumental in representation theory, and these proceedings include surveys on geometric as well as combinatorial constructions of the crystal basis for representations of quantum groups. Humphreys' paper outlines intricate connections among irreducible representations of certain blocks of reduced enveloping algebras of semi-simple Lie algebras in positive characteristic, left cells in two-sided cells of affine Weyl groups, and the geometry of the nilpotent orbits. All of these papers provide the reader with a broad picture of the interaction of many different research areas and should be helpful to those who want to have a glimpse of current research involving representation theory.

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