# CONTEMPORARY MATHEMATICS 

# Primes and Knots 

Toshitake Kohno<br>Masanori Morishita Editors

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# Contemporary Mathematics 

## 416

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Toshitake Kohno<br>Masanori Morishita<br>Editors



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## Preface

The fifteenth year program of the Japan-U.S. Mathematics Institute was entitled "Primes and Knots" and devoted to the connection between algebraic number theory and low-dimensional topology. The program culminated in two conferences, one held as a special session at the 109th AMS annual meeting in Baltimore on January 15 and 16, 2003, and another held as a week-long workshop at the Johns Hopkins University from March 7 to 16, 2003. The conferences were organized by Stavros Garoufalidis, Toshitake Kohno, Jack Morava, Masanori Morishita, and Steve Zucker. The theme of the conferences was the interaction between number theory and topology.

The conferences focused on the following topics:

1. Analogies between primes and knots, number rings and 3-manifolds
2. Avant-garde Galois theory-Galois representations, GrothendieckTeichmüller groups
3. Classical and quantum topology of knots
4. Arithmetic non-commutative geometry and related mathematical physics.

The papers assembled for this volume include speakers' contributions which present extended or modified versions of the lectures delivered at the meetings.

The organizers and editors are grateful to the Department of Mathematics of the Johns Hopkins University and the National Science Foundation for their financial support. Thanks are also due to Professor Kenji Fukaya for his support by Grants-in-Aid for Scientific Research 13852001, Japan Society for the Promotion of Science.

We are pleased to celebrate that in March 2006 the Japan-U.S. Mathematics Institute was awarded the Mathematical Society of Japan's Seki Takakazu Prize for promoting cooperation between the two countries in mathematical research.

March 2006
Toshitake Kohno
Masanori Morishita

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This volume deals systematically with connections between algebraic number theory and low-dimensional topology. Of particular note are various inspiring interactions between number theory and low-dimensional topology discussed in most papers in this volume. For example, quite interesting are the use of arithmetic methods in knot theory and the use of topological methods in Galois theory. Also, expository papers in both number theory and topology included in the volume can help a wide group of readers to understand both fields as well as the interesting analogies and relations that bring them together.

