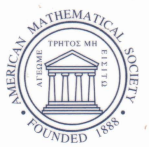
CONTEMPORARY MATHEMATICS

434

Geometric and Topological Methods for Quantum Field Theory

Summer School on Geometric and Topological Methods for Quantum Field Theory
July 11–29, 2005
Villa de Leyva, Colombia

S. Paycha
B. Uribe
Editors



American Mathematical Society

http://dx.doi.org/10.1090/conm/434

Geometric and Topological Methods for Quantum Field Theory

CONTEMPORARY MATHEMATICS

434

Geometric and Topological Methods for Quantum Field Theory

Summer School on Geometric and Topological Methods for Quantum Field Theory
July 11–29, 2005
Villa de Leyva, Colombia

S. Paycha
B. Uribe
Editors

American Mathematical Society
Providence, Rhode Island

2000 *Mathematics Subject Classification.* Primary 53–06, 55–06, 58–06, 81–06.

Library of Congress Cataloging-in-Publication Data

Summer School on Geometric and Topological Methods for Quantum Field Theory (2005 ; Villa de Leyva, Colombia)

Geometric and topological methods for quantum field theory : Summer School on Geometric and Topological Methods for Quantum Field Theory, July 11–29, 2005, Villa de Leyva, Colombia / S. Paycha, B. Uribe, editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; v. 434)

Includes bibliographical references.

ISBN 978-0-8218-4062-7 (alk. paper)

1. Quantum field theory—Congresses. 2. Algebraic topology—Congresses. 3. Geometry, Algebraic—Congresses. I. Paycha, Sylvie. II. Uribe, B. (Bernardo), 1975– III. Title.

QC174.45.A1 S86 2005
530.14′3—dc22 2007060762

---

10 9 8 7 6 5 4 3 2 1 12 11 10 09 08 07

Contents

Introduction

This volume offers an introduction to some recent developments in several active topics at the interface between geometry, topology and quantum field theory:

- Anomalies and noncommutative geometry
- Deformation quantization and Poisson algebras
- Topological quantum field theory and orbifolds

It is based on lectures and short communications delivered during a summer school on "Geometric and Topological Methods for Quantum Field Theory" held in Villa de Leyva, Colombia in July 2005.

The invited lectures, aimed at graduate students in physics or mathematics, start with introductory material before presenting more advanced results. Each lecture is self-contained and can be read independently of the rest.

The volume begins with an introductory course on deformation quantization, in which Martin Bordemann provides a sketch of partial results on the theory of morphisms and modules in that framework in relation to Poisson geometry. Yoshiaki Maeda and his coauthors describe later in this volume a deformation quantization of (infinite dimensional) Fréchet-Poisson algebras associated with Heisenberg Lie algebras.

The reader is led into the realm of noncommutative geometry and its interactions with physics with an introductory lecture by Denis Perrot on anomalies in noncommutative index theory. The author introduces the basic methods of cyclic cohomology and explains the noncommutative counterpart of the Atiyah-Singer index theorem in order to describe anomalies in quantum field theory. A further lecture by Giovanni Landi provides a pedagogical introduction to recent work on deformation of spaces (e.g. toric noncommutative manifolds) and of vector bundles over these spaces, which are relevant both in mathematics and in physics, notably for monopoles and instanton bundles.

Topological quantum field theory and orbifolds are the subject of a lecture by Ernesto Lupercio and Bernardo Uribe, which is a survey on the construction of topological quantum field theories originating from orbifolds such as Chen-Ruan cohomology and orbifold string topology that provides an accessible introduction to these topics for both physicists and geometers.

The invited lectures are followed by nine short communications on a wide spectrum of topics on the borderline of mathematics and physics ranging from quasicristals to invariant instantons through black holes and involving a manifold of mathematical tools borrowed from geometry, algebra and analysis.

We hope that these contributions will give – as much as the school itself seems

to have given – young students the desire to pursue what might be their first acquaintance with some of the problems on the edge of mathematics and physics presented here. On the other hand, we hope that the more advanced reader will find some pleasure in reading about different outlooks on related topics and seeing how the well-known geometric tools prove to be very useful in some areas of quantum field theory.

We are indebted to various organizations for their financial support for this school. Let us first of all thank the Clay Institute in Boston without which this school would not have taken place. We are also deeply grateful to the ICTP in Trieste, for its constant financial support over the years and specifically for this school. We also acknowledge the CDE-IMU for its support. We are also greatly indebted to the Universidad of Los Andes that has been supporting this and many other schools of this kind we have been organizing in Colombia since 1999. Other organizations such as CLAF in Brazil, Colciencias, ICETEX and ICFES in Colombia also contributed in a substantial way to the financial support needed for this school.

Special thanks to Sergio Adarve (Universidad de Los Andes) and Hernán Ocampo (Universidad del Valle), coorganizers of the school, who dedicated time and energy to make this school possible in a country like Colombia, where many difficulties are bound to arise along the way due to social, political and economic problems.

We are also very grateful to Juana Vall-Serra who did a great job for the practical organization of the school, the quality of which was very much appreciated by participants and lecturers. We are also very indebted to Mateo Adarve, Paola Adarve, Marta Casas, Magdalena Cubides, María Inés Cubides, Rodrigo Escobar, Anderson García, Andrés García, Marta Kovacics, Mauricio Morales, Alexandra Parra, Celio Sierra-Paycha and Oriel Sierra-Paycha, for their help in various essential tasks needed for the successful development of the school and to Alexander Cardona, for his permanent assistance and collaboration in the academic organization of the school.

We also would like to thank the administrative staff at the Universidad de los Andes, particularly José Rafael Toro, Vice-rector, Rolando Roldán, Dean of the Science School, Carlos Montenegro, Director of the Mathematics Department, and Bernardo Gómez, Director of the Physics Department, for their constant encouragement and support.

Without the people named here, all of whom helped in the organization in some way or another, before, during and after the school, this scientific event would not have left such vivid memories in the lecturers' and participants' minds. Last but not least, thanks to all the participants who gave us all, lecturers and editors, the impulse to prepare this volume through the enthusiasm they showed during the school, and thank you to all the contributors and referees for their participation in the realization of these proceedings.

The editors,
Sylvie Paycha and Bernardo Uribe

List of Participants

Adarve, Sergio
Uniandes , Colombia

Afanador, Nicolás
Uniandes , Colombia

Anabalón, Andrés
CECS , Chile

Ángel, Jairo Andrés
Stanford , USA

Ángel, Mauricio
UCV , Venezuela

Angueyra, Gina
Uniandes , Colombia

Backelin, Erik
Uniandes , Colombia

Bastidas, Víctor Manuel
Univalle , Colombia

Benavides, Carlos L.
Uniandes , Colombia

Bonilla, Lina del Pilar
Univalle , Colombia

Bonilla, María del Pilar
Univalle , Colombia

Bordemann, Martin
Mulhouse , France

Cáceres, Alejandro
Cambridge , UK

Caicedo, José Alexander
UNAM , Mexico

Cano, Leonardo
MPI Bonn , Germany

Cardona, Alexander
Uniandes , Colombia

Cardona, Carlos Andrés
Univalle , Colombia

Castillo, Edmundo
UCV , Venezuela

Castillo, Óscar Alberto
CECS , Chile

Castro, Darío Alberto
UNAL , Colombia

Caycedo, Felipe
Uniandes , Colombia

Cely, Jorge Enrique
Uniandes , Colombia

Cuberos, Andrés M.
Uniandes , Colombia

Cuervo, William
UNAM , Mexico

Domínguez, Fabio
Uniandes , Colombia

Ellwood, David
CMI , USA

Ferrero, Alejandro
Uniandes , Colombia

Gallardo, Patricio Iván
Univalle , Colombia

Gaviria, Paola Andrea
USP , Brazil

Giniatoulline, Andrei
Uniandes , Colombia

Gómez, Humberto
Univalle , Colombia

Gómez, María Paula
Paris VII , France

Hernández, Francisco J.
UNAM , Mexico

Holguín, Sergio Andrés
UDEA , Colombia

Landi, Giovanni
Trieste , Italy

Larraín, Andrés
Uniandes , Colombia

Lesmes, Jaime
Uniandes , Colombia

Loaiza, Edwin
Univalle , Colombia

Lupercio, Ernesto
CINVESTAV , Mexico

Maeda, Yoshiaki
Keio , Japan

Martínez, Hermes J.
Uniandes , Colombia

Mostovoy, Jacob
UNAM , Mexico

Muñiz, Richard
CIMAT , Mexico

Muñoz, Jhovanny
Univalle , Colombia

Neira, Carolina
Uniandes , Colombia

Ocampo, Hernán
Univalle , Colombia

Oliva, Julio Eduardo
CECS , Chile

Osorio, Cristhian Said
Univalle , Colombia

Palacio, Jorge Humberto
Uniandes , Colombia

Pando Zayas, Leopoldo
Michigan , USA

Pariguán, Eddy J.
UCV , Venezuela

Paycha, Sylvie
Clermont , France

Paz, Gerardo Andrés
Univalle , Colombia

Pérez, Alicia
Uniandes , Colombia

Perrot, Denis
Lyon , France

Plazas, Jorge Andrés
MPI Bonn , Germany

Polesello, Pietro
Padova , Italy

Riveros, David Ricardo
Uniandes , Colombia

Roa, Alexis
Univalle , Colombia

Rodríguez, José del C.
Uniandes , Colombia

Roldán, J. M. Rolando
Uniandes , Colombia

Ryom-Hansen, Steen
UTAL , Chile

Sánchez, José Antonio
UNAL , Colombia

Solarte, Luis Miguel
Univalle , Colombia

Suazo, Erwin
Arizona St. , USA

Uribe, Bernardo
Uniandes , Colombia

Varghese, Mathai
Adelaide , Australia

Ypma, Fonger
Oxford , UK

Zalamea, Iván
Uniandes , Colombia

Titles in This Series

434 **S. Paycha and B. Uribe, Editors,** Geometric and topological methods for quantum field theory, 2007

432 **Dick Canery, Jane Gilman, Juha Heinoren, and Howard Masur, Editors,** In the tradition of Ahlfors-Bers, IV, 2007

431 **Michael Batanin, Alexei Davydov, Michael Johnson, Stephen Lack, and Amnon Neeman, Editors,** Categories in algebra, geometry and mathematical physics, 2007

430 **Idris Assani, Editor,** Ergodic theory and related fields, 2007

429 **Gui-Qiang Chen, Elton Hsu, and Mark Pinsky, Editors,** Stochastic analysis and partial differential equations, 2007

428 **Estela A. Gavosto, Marianne K. Korten, Charles N. Moore, and Rodolfo H. Torres, Editors,** Harmonic analysis, partial differential equations, and related topics, 2007

427 **Anastasios Mallios and Marina Haralampidou, Editors,** Topological algebras and applications, 2007

426 **Fabio Ancona, Irena Lasiecka, Walter Littman, and Roberto Triggiani, Editors,** Control methods in PDE-dynamical systems, 2007

425 **Su Gao, Steve Jackson, and Yi Zhang, Editors,** Advances in Logic, 2007

424 **V. I. Burenko, T. Iwaniec, and S. K. Vodopyanov, Editors,** Analysis and geometry in their interaction, 2007

423 **Christos A. Athanasiadis, Victor V. Batyrev, Dimitrios I. Dais, Martin Henk, and Francisco Santos, Editors,** Algebraic and geometric combinatorics, 2007

422 **JongHae Keum and Shigeyuki Kondō, Editors,** Algebraic Geometry, 2007

421 **Benjamin Fine, Anthony M. Gaglione, and Dennis Spellman, Editors,** Combinatorial group theory, discrete groups, and number theory, 2007

420 **William Chin, James Osterburg, and Declan Quinn, Editors,** Groups, rings and algebras, 2006

419 **Dinh V. Huynh, S. K. Jain, and S. R. López-Permouth, Editors,** Algebra and Its applications, 2006

418 **Lothar Gerritzen, Dorian Goldfeld, Martin Kreuzer, Gerhard Rosenberger, and Vladimir Shpilrain, Editors,** Algebraic methods in cryptography, 2006

417 **Vadim B. Kuznetsov and Siddhartha Sahi, Editors,** Jack, Hall-Littlewood and Macdonald polynomials, 2006

416 **Toshitake Kohno and Masanori Morishita, Editors,** Primes and knots, 2006

415 **Gregory Berkolaiko, Robert Carlson, Stephen A. Fulling, and Peter Kuchment, Editors,** Quantum graphs and their applications, 2006

414 **Deguang Han, Palle E. T. Jorgensen, and David Royal Larson, Editors,** Operator theory, operator algebras, and applications, 2006

413 **Georgia M. Benkart, Jens C. Jantzen, Zongzhu Lin, Daniel K. Nakano, and Brian J. Parshall, Editors,** Representations of algebraic groups, quantum groups and Lie algebras, 2006

412 **Nikolai Chernov, Yulia Karpeshina, Ian W. Knowles, Roger T. Lewis, and Rudi Weikard, Editors,** Recent advances in differential equations and mathematical physics, 2006

411 **J. Marshall Ash and Roger L. Jones, Editors,** Harmonic analysis: Calderón-Zygmund and beyond, 2006

410 **Abba Gumel, Carlos Castillo-Chavez, Ronald E. Mickens, and Dominic P. Clemence, Editors,** Mathematical studies on human disease dynamics: Emerging paradigms and challenges, 2006

409 **Juan Luis Vázquez, Xavier Cabré, and José Antonio Carrillo, Editors,** Recent trends in partial differential equations, 2006

For a complete list of titles in this series, visit the AMS Bookstore at **www.ams.org/bookstore/.**

This volume, based on lectures and short communications at a summer school in Villa de Leyva, Colombia (July 2005), offers an introduction to some recent developments in several active topics at the interface between geometry, topology and quantum field theory. It is aimed at graduate students in physics or mathematics who might want insight in the following topics (covered in five survey lectures):

- Anomalies and noncommutative geometry,
- Deformation quantization and Poisson algebras,
- Topological quantum field theory and orbifolds.

These lectures are followed by nine articles on various topics at the borderline of mathematics and physics ranging from quasicrystals to invariant instantons through black holes, and involving a number of mathematical tools borrowed from geometry, algebra and analysis.

ISBN 978-0-8218-4062-7

9 780821 840627

CONM/434