

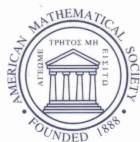
# CONTEMPORARY MATHEMATICS

434

## Geometric and Topological Methods for Quantum Field Theory

Summer School on Geometric and Topological Methods  
for Quantum Field Theory  
July 11–29, 2005  
Villa de Leyva, Colombia

S. Paycha  
B. Uribe  
Editors



# Geometric and Topological Methods for Quantum Field Theory

# CONTEMPORARY MATHEMATICS

---

434

## Geometric and Topological Methods for Quantum Field Theory

Summer School on Geometric and Topological  
Methods for Quantum Field Theory  
July 11–29, 2005  
Villa de Leyva, Colombia

S. Paycha  
B. Uribe  
Editors



---

American Mathematical Society  
Providence, Rhode Island

## Editorial Board

Dennis DeTurck, managing editor

George Andrews   Andreas Blass   Abel Klein

2000 *Mathematics Subject Classification*. Primary 53–06, 55–06, 58–06, 81–06.

---

### Library of Congress Cataloging-in-Publication Data

Summer School on Geometric and Topological Methods for Quantum Field Theory (2005 ; Villa de Leyva, Colombia)

Geometric and topological methods for quantum field theory : Summer School on Geometric and Topological Methods for Quantum Field Theory, July 11–29, 2005, Villa de Leyva, Colombia / S. Paycha, B. Uribe, editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; v. 434)

Includes bibliographical references.

ISBN 978-0-8218-4062-7 (alk. paper)

1. Quantum field theory—Congresses. 2. Algebraic topology—Congresses. 3. Geometry, Algebraic—Congresses. I. Paycha, Sylvie. II. Uribe, B. (Bernardo), 1975– III. Title.

QC174.45.A1 S86 2005

530.14'3—dc22

2007060762

---

**Copying and reprinting.** Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to [reprint-permission@ams.org](mailto:reprint-permission@ams.org).

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2007 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights  
except those granted to the United States Government.

Copyright of individual articles may revert to the public domain 28 years  
after publication. Contact the AMS for copyright status of individual articles.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines  
established to ensure permanence and durability.

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1      12 11 10 09 08 07

## Contents

Introduction	vii
List of participants	ix
<b>Invited Lecturers</b>	
Deformation Quantization: a Mini-Lecture MARTIN BORDEMANN	3
Examples of noncommutative instantons GIOVANNI LANDI	39
Topological Quantum Field Theories, Strings and Orbifolds ERNESTO LUPERCIO AND BERNARDO URIBE	73
Non-formal deformation quantization of Fréchet-Poisson algebras: The Heisenberg Lie algebra case HIDEKI OMORI, YOSHIKI MAEDA, NAOYA MIYAZAKI AND AKIRA YOSHIOKA	99
Anomalies and Noncommutative Index Theory DENIS PERROT	125
<b>Contributed talks</b>	
N-flat connections MAURICIO ÁNGEL AND RAFAEL DÍAZ	163
Dirac Equation in a Black Hole Background ALEJANDRO CÁCERES	173
Homological matrices EDMUNDO CASTILLO AND RAFAEL DÍAZ	181
On Some Qualitative Properties of Stratified Flows ANDREI GINIATOULLINE AND OSWALDO ZAPATA	193
Property (T) and tensor products by irreducible finite dimensional representations for $SL_n(\mathbb{R})$ $n \geq 3$ MARIA PAULA GOMEZ-APARICIO	205
Painlevé VI equation from invariant instantons RICHARD MUÑIZ MANASLISKI	215

Quantum Statistical Mechanics and Class Field Theory JORGE PLAZAS	223
Uniqueness of Kashiwara's quantization of complex contact manifolds PIETRO POLESELLO	233
K-theoretic Gap labeling for Quasicrystals FONGER YPMA	247

## Introduction

This volume offers an introduction to some recent developments in several active topics at the interface between geometry, topology and quantum field theory:

- Anomalies and noncommutative geometry
- Deformation quantization and Poisson algebras
- Topological quantum field theory and orbifolds

It is based on lectures and short communications delivered during a summer school on “Geometric and Topological Methods for Quantum Field Theory” held in Villa de Leyva, Colombia in July 2005.

The invited lectures, aimed at graduate students in physics or mathematics, start with introductory material before presenting more advanced results. Each lecture is self-contained and can be read independently of the rest.

The volume begins with an introductory course on deformation quantization, in which Martin Bordemann provides a sketch of partial results on the theory of morphisms and modules in that framework in relation to Poisson geometry. Yoshiaki Maeda and his coauthors describe later in this volume a deformation quantization of (infinite dimensional) Fréchet-Poisson algebras associated with Heisenberg Lie algebras.

The reader is led into the realm of noncommutative geometry and its interactions with physics with an introductory lecture by Denis Perrot on anomalies in noncommutative index theory. The author introduces the basic methods of cyclic cohomology and explains the noncommutative counterpart of the Atiyah-Singer index theorem in order to describe anomalies in quantum field theory. A further lecture by Giovanni Landi provides a pedagogical introduction to recent work on deformation of spaces (e.g. toric noncommutative manifolds) and of vector bundles over these spaces, which are relevant both in mathematics and in physics, notably for monopoles and instanton bundles.

Topological quantum field theory and orbifolds are the subject of a lecture by Ernesto Lupercio and Bernardo Uribe, which is a survey on the construction of topological quantum field theories originating from orbifolds such as Chen-Ruan cohomology and orbifold string topology that provides an accessible introduction to these topics for both physicists and geometers.

The invited lectures are followed by nine short communications on a wide spectrum of topics on the borderline of mathematics and physics ranging from quasicrystals to invariant instantons through black holes and involving a manifold of mathematical tools borrowed from geometry, algebra and analysis.

We hope that these contributions will give – as much as the school itself seems

to have given – young students the desire to pursue what might be their first acquaintance with some of the problems on the edge of mathematics and physics presented here. On the other hand, we hope that the more advanced reader will find some pleasure in reading about different outlooks on related topics and seeing how the well-known geometric tools prove to be very useful in some areas of quantum field theory.

We are indebted to various organizations for their financial support for this school. Let us first of all thank the Clay Institute in Boston without which this school would not have taken place. We are also deeply grateful to the ICTP in Trieste, for its constant financial support over the years and specifically for this school. We also acknowledge the CDE-IMU for its support. We are also greatly indebted to the Universidad de Los Andes that has been supporting this and many other schools of this kind we have been organizing in Colombia since 1999. Other organizations such as CLAF in Brazil, Colciencias, ICETEX and ICFES in Colombia also contributed in a substantial way to the financial support needed for this school.

Special thanks to Sergio Adarve (Universidad de Los Andes) and Hernán Ocampo (Universidad del Valle), coorganizers of the school, who dedicated time and energy to make this school possible in a country like Colombia, where many difficulties are bound to arise along the way due to social, political and economic problems.

We are also very grateful to Juana Vall-Serra who did a great job for the practical organization of the school, the quality of which was very much appreciated by participants and lecturers. We are also very indebted to Mateo Adarve, Paola Adarve, Marta Casas, Magdalena Cubides, María Inés Cubides, Rodrigo Escobar, Anderson García, Andrés García, Marta Kovacics, Mauricio Morales, Alexandra Parra, Celio Sierra-Paycha and Oriel Sierra-Paycha, for their help in various essential tasks needed for the successful development of the school and to Alexander Cardona, for his permanent assistance and collaboration in the academic organization of the school.

We also would like to thank the administrative staff at the Universidad de los Andes, particularly José Rafael Toro, Vice-rector, Rolando Roldán, Dean of the Science School, Carlos Montenegro, Director of the Mathematics Department, and Bernardo Gómez, Director of the Physics Department, for their constant encouragement and support.

Without the people named here, all of whom helped in the organization in some way or another, before, during and after the school, this scientific event would not have left such vivid memories in the lecturers' and participants' minds. Last but not least, thanks to all the participants who gave us all, lecturers and editors, the impulse to prepare this volume through the enthusiasm they showed during the school, and thank you to all the contributors and referees for their participation in the realization of these proceedings.

The editors,  
Sylvie Paycha and Bernardo Uribe



## List of Participants

Adarve, Sergio  
Uniandes , Colombia

Afanador, Nicolás  
Uniandes , Colombia

Anabalón, Andrés  
CECS , Chile

Ángel, Jairo Andrés  
Stanford , USA

Ángel, Mauricio  
UCV , Venezuela

Angueyra, Gina  
Uniandes , Colombia

Backelin, Erik  
Uniandes , Colombia

Bastidas, Víctor Manuel  
Univalle , Colombia

Benavides, Carlos L.  
Uniandes , Colombia

Bonilla, Lina del Pilar  
Univalle , Colombia

Bonilla, María del Pilar  
Univalle , Colombia

Bordemann, Martin  
Mulhouse , France

Cáceres, Alejandro  
Cambridge , UK

Caicedo, José Alexander  
UNAM , Mexico

Cano, Leonardo  
MPI Bonn , Germany

Cardona, Alexander  
Uniandes , Colombia

Cardona, Carlos Andrés  
Univalle , Colombia

Castillo, Edmundo  
UCV , Venezuela

Castillo, Óscar Alberto  
CECS , Chile

Castro, Darío Alberto  
UNAL , Colombia

Caycedo, Felipe  
Uniandes , Colombia

Cely, Jorge Enrique  
Uniandes , Colombia

Cuberos, Andrés M.  
Uniandes , Colombia

Cuervo, William  
UNAM , Mexico

Domínguez, Fabio  
Uniandes , Colombia

Ellwood, David  
CMI , USA

Ferrero, Alejandro  
Uniandes , Colombia

Gallardo, Patricio Iván  
Univalle , Colombia

Gaviria, Paola Andrea  
USP , Brazil

Giniatoulline, Andrei  
Uniandes , Colombia

Gómez, Humberto  
Univalle , Colombia

Gómez, María Paula  
Paris VII , France

Hernández, Francisco J.  
UNAM , Mexico

Holguín, Sergio Andrés  
UDEA , Colombia

Landi, Giovanni  
Trieste , Italy

Larraín, Andrés  
Uniandes , Colombia

Lesmes, Jaime  
Uniandes , Colombia

Loaiza, Edwin  
Univalle , Colombia

Lupercio, Ernesto  
CINVESTAV , Mexico

Maeda, Yoshiaki  
Keio , Japan

Martínez, Hermes J.  
Uniandes , Colombia

Mostovoy, Jacob  
UNAM , Mexico

Muñiz, Richard  
CIMAT , Mexico

Muñoz, Jhovanny  
Univalle , Colombia

Neira, Carolina  
Uniandes , Colombia

Ocampo, Hernán  
Univalle , Colombia

Oliva, Julio Eduardo  
CECS , Chile

Osorio, Cristhian Said  
Univalle , Colombia

Palacio, Jorge Humberto  
Uniandes , Colombia

Pando Zayas, Leopoldo  
Michigan , USA

Pariguán, Eddy J.  
UCV , Venezuela

Paycha, Sylvie  
Clermont , France

Paz, Gerardo Andrés  
Univalle , Colombia

Pérez, Alicia  
Uniandes , Colombia

Perrot, Denis  
Lyon , France

Plazas, Jorge Andrés  
MPI Bonn , Germany

Polesello, Pietro  
Padova , Italy

Riveros, David Ricardo  
Uniandes , Colombia

Roa, Alexis  
Univalle , Colombia

Rodríguez, José del C.  
Uniandes , Colombia

Roldán, J. M. Rolando  
Uniandes , Colombia

Ryom-Hansen, Steen  
UTAL , Chile

Sánchez, José Antonio  
UNAL , Colombia

Solarte, Luis Miguel  
Univalle , Colombia

Suazo, Erwin  
Arizona St. , USA

Uribe, Bernardo  
Uniandes , Colombia

Varghese, Mathai  
Adelaide , Australia

Ypma, Fonger  
Oxford , UK

Zalamea, Iván  
Uniandes , Colombia

## Titles in This Series

- 434 **S. Paycha and B. Uribe, Editors**, Geometric and topological methods for quantum field theory, 2007
- 432 **Dick Canary, Jane Gilman, Juha Heinonen, and Howard Masur, Editors**, In the tradition of Ahlfors-Bers, IV, 2007
- 431 **Michael Batanin, Alexei Davydov, Michael Johnson, Stephen Lack, and Amnon Neeman, Editors**, Categories in algebra, geometry and mathematical physics, 2007
- 430 **Idris Assani, Editor**, Ergodic theory and related fields, 2007
- 429 **Gui-Qiang Chen, Elton Hsu, and Mark Pinsky, Editors**, Stochastic analysis and partial differential equations, 2007
- 428 **Estela A. Gavosto, Marianne K. Kortén, Charles N. Moore, and Rodolfo H. Torres, Editors**, Harmonic analysis, partial differential equations, and related topics, 2007
- 427 **Anastasios Mallios and Marina Haralampidou, Editors**, Topological algebras and applications, 2007
- 426 **Fabio Ancona, Irena Lasiecka, Walter Littman, and Roberto Triggiani, Editors**, Control methods in PDE-dynamical systems, 2007
- 425 **Su Gao, Steve Jackson, and Yi Zhang, Editors**, Advances in Logic, 2007
- 424 **V. I. Burenko, T. Iwaniec, and S. K. Vodopyanov, Editors**, Analysis and geometry in their interaction, 2007
- 423 **Christos A. Athanasiadis, Victor V. Batyrev, Dimitrios I. Dais, Martin Henk, and Francisco Santos, Editors**, Algebraic and geometric combinatorics, 2007
- 422 **JongHae Keum and Shigeyuki Kondō, Editors**, Algebraic Geometry, 2007
- 421 **Benjamin Fine, Anthony M. Gaglione, and Dennis Spellman, Editors**, Combinatorial group theory, discrete groups, and number theory, 2007
- 420 **William Chin, James Osterburg, and Declan Quinn, Editors**, Groups, rings and algebras, 2006
- 419 **Dinh V. Huynh, S. K. Jain, and S. R. López-Permouth, Editors**, Algebra and Its applications, 2006
- 418 **Lothar Gerritzen, Dorian Goldfeld, Martin Kreuzer, Gerhard Rosenberger, and Vladimir Shpilrain, Editors**, Algebraic methods in cryptography, 2006
- 417 **Vadim B. Kuznetsov and Siddhartha Sahi, Editors**, Jack, Hall-Littlewood and Macdonald polynomials, 2006
- 416 **Toshitake Kohno and Masanori Morishita, Editors**, Primes and knots, 2006
- 415 **Gregory Berkolaiko, Robert Carlson, Stephen A. Fulling, and Peter Kuchment, Editors**, Quantum graphs and their applications, 2006
- 414 **Deguang Han, Palle E. T. Jorgensen, and David Royal Larson, Editors**, Operator theory, operator algebras, and applications, 2006
- 413 **Georgia M. Benkart, Jens C. Jantzen, Zongzhu Lin, Daniel K. Nakano, and Brian J. Parshall, Editors**, Representations of algebraic groups, quantum groups and Lie algebras, 2006
- 412 **Nikolai Chernov, Yulia Karpeshina, Ian W. Knowles, Roger T. Lewis, and Rudi Weikard, Editors**, Recent advances in differential equations and mathematical physics, 2006
- 411 **J. Marshall Ash and Roger L. Jones, Editors**, Harmonic analysis: Calderón-Zygmund and beyond, 2006
- 410 **Abba Gumel, Carlos Castillo-Chavez, Ronald E. Mickens, and Dominic P. Clemence, Editors**, Mathematical studies on human disease dynamics: Emerging paradigms and challenges, 2006
- 409 **Juan Luis Vázquez, Xavier Cabré, and José Antonio Carrillo, Editors**, Recent trends in partial differential equations, 2006

TITLES IN THIS SERIES

- 408 **Habib Ammari and Hyeonbae Kang, Editors**, Inverse problems, multi-scale analysis and effective medium theory, 2006
- 407 **Alejandro Ádem, Jesús González, and Guillermo Pastor, Editors**, Recent developments in algebraic topology, 2006
- 406 **José A. de la Peña and Raymundo Bautista, Editors**, Trends in representation theory of algebras and related topics, 2006
- 405 **Andrew Markoe and Eric Todd Quinto, Editors**, Integral geometry and tomography, 2006
- 404 **Alexander Borichev, Håkan Hedenmalm, and Kehe Zhu, Editors**, Bergman spaces and related topics in complex analysis, 2006
- 403 **Tyler J. Jarvis, Takashi Kimura, and Arkady Vaintrob, Editors**, Gromov-Witten theory of spin curves and orbifolds, 2006
- 402 **Zvi Arad, Mariagrazia Bianchi, Wolfgang Herfort, Patrizia Longobardi, Mercede Maj, and Carlo Scoppola, Editors**, Ischia group theory 2004, 2006
- 401 **Katrin Becker, Melanie Becker, Aaron Bertram, Paul S. Green, and Benjamin McKay, Editors**, Snowbird lectures on string geometry, 2006
- 400 **Shiferaw Berhanu, Hua Chen, Jorge Hounie, Xiaojun Huang, Sheng-Li Tan, and Stephen S.-T. Yau, Editors**, Recent progress on some problems in several complex variables and partial differential equations, 2006
- 399 **Dominique Arlettaz and Kathryn Hess, Editors**, An Alpine anthology of homotopy theory, 2006
- 398 **Jay Jorgenson and Lynne Walling, Editors**, The ubiquitous heat kernel, 2006
- 397 **José M. Muñoz Porras, Sorin Popescu, and Rubí E. Rodríguez, Editors**, The geometry of Riemann surfaces and Abelian varieties, 2006
- 396 **Robert L. Devaney and Linda Keen, Editors**, Complex dynamics: Twenty-five years after the appearance of the Mandelbrot set, 2006
- 395 **Gary R. Jensen and Steven G. Krantz, Editors**, 150 Years of Mathematics at Washington University in St. Louis, 2006
- 394 **Rostislav Grigorchuk, Michael Mihalik, Mark Sapir, and Zoran Šuník, Editors**, Topological and asymptotic aspects of group theory, 2006
- 393 **Alec L. Matheson, Michael I. Stessin, and Richard M. Timoney, Editors**, Recent advances in operator-related function theory, 2006
- 392 **Stephen Berman, Brian Parshall, Leonard Scott, and Weiqiang Wang, Editors**, Infinite-dimensional aspects of representation theory and applications, 2005
- 391 **Jürgen Fuchs, Jouko Mickelsson, Grigori Rozenblioum, Alexander Stolin, and Anders Westerberg, Editors**, Noncommutative geometry and representation theory in mathematical physics, 2005
- 390 **Sudhir Ghorpade, Hema Srinivasan, and Jugal Verma, Editors**, Commutative algebra and algebraic geometry, 2005
- 389 **James Eells, Etienne Ghys, Mikhail Lyubich, Jacob Palis, and José Seade, Editors**, Geometry and dynamics, 2005
- 388 **Ravi Vakil, Editor**, Snowbird lectures in algebraic geometry, 2005
- 387 **Michael Entov, Yehuda Pinchover, and Michah Sageev, Editors**, Geometry, spectral theory, groups, and dynamics, 2005
- 386 **Yasuyuki Kachi, S. B. Mulay, and Pavlos Tzermias, Editors**, Recent progress in arithmetic and algebraic geometry, 2005

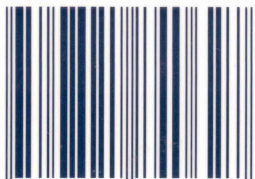
For a complete list of titles in this series, visit the  
AMS Bookstore at [www.ams.org/bookstore/](http://www.ams.org/bookstore/).

This volume, based on lectures and short communications at a summer school in Villa de Leyva, Colombia (July 2005), offers an introduction to some recent developments in several active topics at the interface between geometry, topology and quantum field theory. It is aimed at graduate students in physics or mathematics who might want insight in the following topics (covered in five survey lectures):

- Anomalies and noncommutative geometry,
- Deformation quantization and Poisson algebras,
- Topological quantum field theory and orbifolds.

These lectures are followed by nine articles on various topics at the borderline of mathematics and physics ranging from quasicrystals to invariant instantons through black holes, and involving a number of mathematical tools borrowed from geometry, algebra and analysis.

ISBN 978-0-8218-4062-7



9 780821 840627

CONM/434

AMS *on the Web*  
[www.ams.org](http://www.ams.org)