

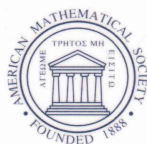
# CONTEMPORARY MATHEMATICS

462

## Non-commutative Geometry in Mathematics and Physics

The XI Solomon Lefschetz Memorial Lecture Series  
and Topics in Deformation Quantization  
and Non-commutative Structures  
September 7–9, 2005  
Mexico City, Mexico

Giuseppe Dito  
Hugo García-Compeán  
Ernesto Lupercio  
Francisco J. Turrubiates  
Editors



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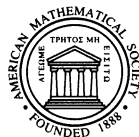
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## Contents

Foreword	vii
XI Solomon Lefschetz Memorial Lecture Series: Hodge structures in non-commutative geometry MAXIM KONTSEVICH	1
Hopfish structure and modules over irrational rotation algebras CHRISTIAN BLOHMANN, XIANG TANG, AND ALAN WEINSTEIN	23
Deformations and quantizations, an introductory overview DANIEL STERNHEIMER	41
SDYM and heavenly equations in deformation quantization SEBASTIAN FORMAŃSKI AND MACIEJ PRZANOWSKI	55
On a possible construction of noncommutative topological invariants O. OBREGÓN, C. RAMÍREZ, AND M. SABIDO	73
On coherent states for spaces of holomorphic functions related to the hydrogen atom problem in dimensions $n = 2, 3, 5$ DANIEL ROJAS-SANDOVAL AND CARLOS VILLEGAS-BLAS	87
Noncommutativity from canonical and noncanonical structures MARCOS ROSENBAUM, J. DAVID VERGARA, AND L. ROMÁN JUÁREZ	105
Commutative algebras of Toeplitz operators and Berezin quantization NIKOLAI VASILEVSKI	125

## Foreword

Since the 19th century, non-commutative structures have played an important role in physics and mathematics. One can mention the work of Galois as an important early example in which non-commutativity greatly informed questions in commutative mathematics. This role expanded considerably with the inception of quantum mechanics in the early years of the 20th century, and since then it only has grown. The late 20th century has seen the advent of non-commutative geometry and a fertile ground of interaction between physics and mathematics.

This volume comprises the contributions of the *XI Solomon Lefschetz Memorial Lectures* and the satellite conference: *Topics in Deformation Quantization and Noncommutative Structures* held at the Centro de Investigación y de Estudios Avanzados (CINVESTAV), Mexico City, Mexico, September 7-9 of 2005. This event was organized in a close collaboration between the departments of Physics and Mathematics with its funding coming from both departments and from various grants funded by the Consejo Nacional de Ciencia y Tecnología (Conacyt).

The Lefschetz Memorial Lectures originated in honor of Prof. Solomon Lefschetz are held periodically in autumn when they spotlight a distinguished mathematician. Prof. Lefschetz is well known for his contributions to mathematics but perhaps it is less known that since 1945 he visited Mexico regularly, jump-starting the Mexican school in topology. His contribution to mathematics in Mexico was, as a result of these visits, of major importance. His contributions were recognized when he was awarded the Order of the Aztec Eagle in 1964. The Mathematics Department of the Cinvestav established the Solomon Lefschetz memorial lectures in 1975 to honor him. A brief history of the lectures can be glimpsed by looking at the impressive list of Lefschetz Lecturers throughout the years:

- (1975) Jack Hale
- (1976) Michael Atiyah
- (1977) Enrico Bombieri
- (1978) Shing S. Chern
- (1979) John Milnor
- (1982) David Mumford
- (1985) William Thurston
- (1989) Ya G. Sinai
- (1991) V. P. Maslov
- (1995) Sergio Albeverio
- (2005) Maxim Kontsevich
- (2006) Dennis Sullivan
- (2007) Graeme Segal

We were very fortunate to have Prof. Maxim Kontsevich (IHES) deliver the lectures for the 30th anniversary of the event, and after a decade that they were suspended due to the economic situation of the country. His contribution to the volume is a historical record of his lectures, which we are sure will become a seminal reference in the field of non-commutative geometry and related areas.

This event and the revival of the lectures would have been impossible without the close collaboration of the departments of Physics and Mathematics. The year 2005 was also the 100th anniversary of Albert Einstein's *annus mirabilis* and thus the 32nd session of the General Conference of UNESCO adopted in November 2003 a resolution supporting the initiative of 2005 as the World Year of Physics. The Physics Department of the Cinvestav under the direction of Dr. Gerardo Contreras wholeheartedly supported the initiative of organizing a satellite conference on Topics in Deformation Quantization and Non-commutative Structures in the framework of the World Year of Physics. This conference benefited from the contribution of two distinguished speakers: Daniel Sternheimer and Alan Weinstein, and numerous other participants. On the side of the Mathematics Department, the revival of the Lefschetz Lectures would not have happened without the whole support of Isidoro Gitler, Chairman of the Department. We are also indebted to the Institut de Mathématiques de Bourgogne at University of Bourgogne for the help and support given during the organization of the entire event as well as to all the members of the organizing committee.

This book consists of 8 papers that reflect the authors' contribution to the conference and the Notes of the Lefschetz Lectures. All the papers were thoroughly refereed by experts in the field to whom we want to thank for their important collaboration. We hope that this book will be valuable for specialists in non-commutative structures and related fields, both in physics as mathematics.

We want to thank the participants (over 200 people) from many universities and institutions of Mexico, France, United States and Colombia who made the lectures and the conference such a success. We are grateful for the financial support of the departments of physics and mathematics of Cinvestav and also to Conacyt. We appreciate the effort of the local committee and all the personnel from Cinvestav. Special thanks go to Samuel Gitler for his scientific expertise and generous funding, Elias Micha for helping with the organization of the conference, and to Jose Luis Enriquez, Anabel Lagos and Daniel Pérez for their technical assistance.

Giuseppe Dito, Hugo García-Compeán, Ernesto Lupercio, Francisco J. Turrubiates

This volume represents the proceedings of the conference on Topics in Deformation Quantization and Non-Commutative Structures held in Mexico City in September 2005. It contains survey papers and original contributions by various experts in the fields of deformation quantization and non-commutative derived algebraic geometry in the interface between mathematics and physics.

It also contains an article based on the XI Solomon Lefschetz Memorial Lectures given by M. Kontsevich, which were delivered as part of the conference.

This is an excellent introductory volume for readers interested in learning about quantization as deformation, Hopf algebras, and Hodge structures in the framework of non-commutative algebraic geometry.

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