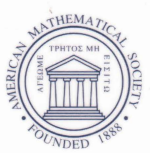
CONTEMPORARY MATHEMATICS

463

Computational Arithmetic Geometry

AMS Special Session
April 29–30, 2006
San Francisco State University
San Francisco, California

Kristin E. Lauter
Kenneth A. Ribet
Editors



American Mathematical Society

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Computational Arithmetic Geometry

Special Session at the San Francisco AMS meeting, April 29-30, 2006.

Front group: Iftikhar Burhanuddin, Tong Hai Yang, Ken Ono, Alina Cojocaru, Kristin Lauter, Kenneth Ribet, René Schoof, Loïc Merel, Peter Stevenhagen

Back group: Everett Howe, David Freeman, Jordan Ellenberg, Kate Stevenson, Jayce Getz, Nils Bruin, Dimitar Jetchev, Bjorn Poonen, David Grant, Denis Charles, Ronald van Luijk, William McCallum, Kiran Kedlaya, David Zywina, Jared Weinstein, William Cherry, David Brown, Lassina Dembele, Byungchul Cha

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Photo courtesy of Kenneth A. Ribet

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To Tom, Joyce, and Josephine
To Lisa, Caroline and Stephanie

Contents

Preface

This volume is the record of the special session on computational aspects of arithmetic geometry at the April, 2006 regional meeting of the American Mathematical Society at San Francisco State University. Our session attracted a large number of outstanding researchers in the field, many of whom presented talks at the session and contributed manuscripts to this volume.

In planning our gathering, we decided to stress computational issues because of recent and ongoing increases in computing power. These increases have enabled computational breakthroughs in any number of areas of arithmetic geometry. For example, Cremona's tables of elliptic curves now go up to conductor 120,000 instead of just conductor 1,000; tables of Hilbert class fields are known for discriminant up to at least 5,000; and special values of Hilbert and Siegel modular forms can be calculated to extremely high precision. In many cases, these experimental capabilities have led to new observations and ideas for progress in the field. They have also led to natural algorithmic questions on the feasibility and efficiency of many computations, especially for the purpose of applications in cryptography.

As our gathering took shape, we were able to identify themes that united clusters of talks: modular abelian varieties; Tate–Shafarevich groups of elliptic curves; Stark's conjectures; Hilbert modular varieties; curves with complex multiplication. In scheduling the lectures, we grouped these clusters together whenever possible.

A thread that ran through a number of the talks was the application of number theory and algebraic geometry to cryptography and coding theory. Potential applications in these areas have suggested interesting mathematical questions and computational challenges that are addressed by articles in this volume. For example, the article by Bach and Charles addresses hardness of computing eigenforms and relates it to breaking RSA cryptosystems based on the hardness of factoring integers. The contribution by Bröker and Stevenhagen addresses the issue of generating elliptic curves for use in cryptography by finding an efficient algorithm for generating good parameters for using the CM (complex multiplication) method. The contribution by Deajim and Grant uncovers interesting number theory questions when constructing good space-time codes. Two papers relate to zeta functions of curves and surfaces, one providing algorithms for computing zeta functions and one answering the question of which zeta functions of supersingular genus 2 curves are possible over finite fields of characteristic 3. We hope that this collection will interest number theorists, algebraic geometers and cryptographers and that its contents will contribute to the ongoing interaction among these groups.

Kristin Lauter
Ken Ribet

Schedule of talks

Saturday morning, April 29:
Heegner points and Tate-Shafarevich Groups

8:00am–8:30am: William McCallum
Yet more nontrivial elements in Shafarevich-Tate groups of Fermat curves.

8:30am–9:00am: Byungchul Cha
Vanishing of some cohomology groups and bounds for Shafarevich–Tate groups of elliptic curves.

9:00am–9:30am: Samit Dasgupta
Shintani zeta-functions and Gross-Stark units for totally real fields.

9:30am–10:00am: Ken McMurdy
Heegner Points on $X_0(p^n)$.

10:00am–10:30am: Nils Bruin
Solvability of small curves of genus 2.

10:30am–11:00am: William Stein
SAGE — Software for Algebra and Geometry Experimentation.

Saturday afternoon, April 29:
Hilbert modular surfaces

3:00pm–3:30pm: Tonghai Yang
A basic intersection problem on Hilbert modular surfaces.

3:30pm–4:00pm: Lassina Dembele
Examples of automorphic forms on the unitary group in three variables.

4:00pm–4:30pm: Amod Agashe
Rational torsion of elliptic curves and the cuspidal subgroup.

4:30pm–5:00pm: Jayce Getz
Hilbert modular forms, intersection homology base change.

5:00pm–5:30pm: Ken Ono
Traces of singular moduli on Hilbert surfaces.

Sunday morning, April 30:
Curves I

8:00am–8:30am: Kiran Kedlaya
Computing zeta functions of surfaces using p-adic cohomology.

8:30am–9:00am: Jeff Achter
Divisibility of function field class numbers.

9:00am–9:30am: Alina Cojocaru
Frobenius fields for Drinfeld modules of rank 2

9:30am–10:00am: Denis Charles
Some applications of the graph of supersingular elliptic curves.

10:00am–10:30am: Everett Howe
Jacobians in isogeny classes of supersingular surfaces over finite fields.

10:30am–11:00am: Peter Stevenhagen
Constructing elliptic curves in almost polynomial time

Sunday afternoon, April 30:
Curves II

3:00pm–3:30pm: Rachel Pries
The p-torsion of curves in characteristic p.

3:30pm–4:00pm: Kate Stevenson
Local Galois theory in dimension two.

4:00pm–4:30pm: David Grant
Number Theoretic Aspects of Space-Time Codes.

4:30pm–5:00pm: Bjorn Poonen
Gonality of modular curves in characteristic p.

5:00pm–5:30pm: Jordan Ellenberg
Reflection principles and l-parts of class groups.

5:30pm–6:00pm: Rene Schoof
Modular curves and semistable abelian varieties over $\mathbb{Q}$.

Titles in This Series

463 **Kristin E. Lauter and Kenneth A. Ribet, Editors,** Computational arithmetic geometry, 2008

462 **Giuseppe Dito, Hugo García-Compeán, Ernesto Lupercio, and Francisco J. Turrubiates, Editors,** Non-commutative geometry in mathematics and physics, 2008

461 **Gary L. Mullen, Daniel Panario, and Igor Shparlinski, Editors,** Finite Fields and Applications, 2008

460 **Megumi Harada, Yael Karshon, Mikiya Masuda, and Taras Panov, Editors,** Toric topology, 2008

459 **Marcelo J. Saia and José Seade, Editors,** Real and complex singularities, 2008

458 **Jinho Baik, Thomas Kriecherbauer, Luen-Chau Li, Kenneth D. T-R McLaughlin, and Carlos Tomei, Editors,** Integrable systems and random matrices, 2008

457 **Tewodros Amdeberhan and Victor H. Moll, Editors,** Tapas in experimental mathematics, 2008

456 **S. K. Jain and S. Parvathi, Editors,** Noncommutative rings, group rings, diagram algebras and their applications, 2008

455 **Mark Agranovsky, Daoud Bshouty, Lavi Karp, Simeon Reich, David Shoikhet, and Lawrence Zalcman, Editors,** Complex analysis and dynamical systems III, 2008

454 **Rita A. Hibschweiler and Thomas H. MacGregor, Editors,** Banach spaces of analytic functions, 2008

453 **Jacob E. Goodman, János Pach, and Richard Pollack, Editors,** Surveys on Discrete and Computational Geometry–Twenty Years Later, 2008

452 **Matthias Beck, Christian Haase, Bruce Reznick, Michèle Vergne, Volkmar Welker, and Ruriko Yoshida, Editors,** Integer points in polyhedra, 2008

451 **David R. Larson, Peter Massopust, Zuhair Nashed, Minh Chuong Nguyen, Manos Papadakis, and Ahmed Zayed, Editors,** Frames and operator theory in analysis and signal processing, 2008

450 **Giuseppe Dito, Jiang-Hua Lu, Yoshiaki Maeda, and Alan Weinstein, Editors,** Poisson geometry in mathematics and physics, 2008

449 **Robert S. Doran, Calvin C. Moore, and Robert J. Zimmer, Editors,** Group representations, ergodic theory, and mathematical physics: A tribute to George W. Mackey, 2007

448 **Alberto Corso, Juan Migliore, and Claudia Polini, Editors,** Algebra, geometry and their interactions, 2007

447 **François Germinet and Peter Hislop, Editors,** Adventures in mathematical physics, 2007

446 **Henri Berestycki, Michiel Bertsch, Felix E. Browder, Louis Nirenberg, Lambertus A. Peletier, and Laurent Véron, Editors,** Perspectives in Nonliner Partial Differential Equations, 2007

445 **Laura De Carli and Mario Milman, Editors,** Interpolation Theory and Applications, 2007

444 **Joseph Rosenblatt, Alexander Stokolos, and Ahmed I. Zayed, Editors,** Topics in harmonic analysis and ergodic theory, 2007

443 **Joseph Stephen Verducci and Xiaotong Shen, Editors,** Prediction and discovery, 2007

442 **Yi-Zhi Huang and Kailash C Misra, Editors,** Lie algebras, vertex operator algbras and their applications, 2007

441 **Louis H. Kauffman, David E. Radford, and Fernando J. O. Souza, Editors,** Hopf algebras and generalizations, 2007

With the recent increase in available computing power, new computations are possible in many areas of arithmetic geometry. To name just a few examples, Cremona's tables of elliptic curves now go up to conductor 120,000 instead of just conductor 1,000, tables of Hilbert class fields are known for discriminant up to at least 5,000, and special values of Hilbert and Siegel modular forms can be calculated to extremely high precision. In many cases, these experimental capabilities have led to new observations and ideas for progress in the field. They have also led to natural algorithmic questions on the feasibility and efficiency of many computations, especially for the purpose of applications in cryptography. The AMS Special Session on Computational Arithmetic Geometry, held on April 29–30, 2006, in San Francisco, CA, gathered together many of the people currently working on the computational and algorithmic aspects of arithmetic geometry. This volume contains research articles related to talks given at the session. The majority of articles are devoted to various aspects of arithmetic geometry, mainly with a computational approach.

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