

# CONTEMPORARY MATHEMATICS

471

## Special Functions and Orthogonal Polynomials

AMS Special Session  
on Special Functions and Orthogonal Polynomials  
April 21–22, 2007  
Tucson, Arizona

Diego Dominici  
Robert S. Maier  
Editors



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2000 *Mathematics Subject Classification*. Primary 33–06.

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### Library of Congress Cataloging-in-Publication Data

Special functions and orthogonal polynomials : AMS Special Session, April 21–22, 2007, Tucson, Arizona / Diego Dominici, Robert S. Maier, editors.

p. cm. — (Contemporary mathematics ISSN 0271-4132 ; v. 471)

Includes bibliographical references.

ISBN 978-0-8218-4650-6 (alk. paper)

1. Functions, Special—Congresses. 2. Orthogonal polynomials—Congresses. I. Dominici, Diego, 1972– II. Maier, Robert Sullivan, 1957–

QA351.S695 2008  
515'.5—dc22

2008022201

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10 9 8 7 6 5 4 3 2 1      13 12 11 10 09 08

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## Preface

This volume contains articles by speakers in the AMS Special Session on Special Functions and Orthogonal Polynomials, held on 21–22 April, 2007 at the campus of the University of Arizona in Tucson. (Details of this special session are available at the AMS website.) The articles cover a wide range of topics in the theory of special functions, and the related theory of orthogonal polynomials. The authors include pure mathematicians, applied mathematicians, and theoretical physicists. A few of the articles have an expository flavor, but original research predominates.

Although there is no formal definition or comprehensive list of “special functions,” most mathematicians, physicists, and engineers will agree on which functions belong to this category. This is because most of the classical special functions arose as solutions of problems in applied mathematics and physics. For this reason, Turán and Askey have suggested calling them “useful functions.” Well-known useful ones include the Bessel functions (useful, e.g., in Kepler’s problem), Legendre polynomials (useful, e.g., in the attraction of ellipsoids), Airy functions (useful, e.g., in the theory of rainbows), parabolic cylinder functions, etc. In recent decades the list has been extended, and special functions have increasingly received the attention of pure mathematicians.

This is the first overview of the modern field of special functions to appear in the Contemporary Mathematics series. All major subfields are represented, including applications to algebraic geometry, the theory of nonlinear waves, and conformal mapping; hypergeometric and  $q$ -hypergeometric series, and theta functions; elliptic functions; combinatorial generating functions; fractional calculus; the symbolic and numerical evaluation of integrals; and asymptotic analysis. Our hope is that the volume gives a true picture of the vitality of the field. This vitality is reflected in the large number of research papers appearing each year, and the increasing number of related conferences.

As co-organizers and editors, we thank all participants and contributors. We are grateful to the American Mathematical Society for assistance in organizing the special session, and in the publication of this volume. We especially thank Christine Thivierge of the AMS staff, for her efficient support in the latter.

Diego Dominici  
Robert S. Maier

This volume contains fourteen articles that represent the AMS Special Session on Special Functions and Orthogonal Polynomials, held in Tucson, Arizona in April of 2007. It gives an overview of the modern field of special functions with all major subfields represented, including: applications to algebraic geometry, asymptotic analysis, conformal mapping, differential equations, elliptic functions, fractional calculus, hypergeometric and  $q$ -hypergeometric series, nonlinear waves, number theory, symbolic and numerical evaluation of integrals, and theta functions. A few articles are expository, with extensive bibliographies, but all contain original research.

This book is intended for pure and applied mathematicians who are interested in recent developments in the theory of special functions. It covers a wide range of active areas of research and demonstrates the vitality of the field.

ISBN 978-0-8218-4650-6



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