

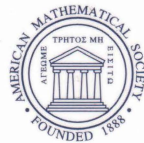
CONTEMPORARY MATHEMATICS

472

Representation Theory of Real Reductive Lie Groups

AMS-IMS-SIAM Joint Summer Research Conference
June 4–8, 2006
Snowbird, Utah

James Arthur
Wilfried Schmid
Peter E. Trapa
Editors



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Preface

The conference *Representation Theory of Real Reductive Lie Groups* took place in Snowbird, Utah, in June of 2006. It followed a preparatory graduate mini-course devoted to the case of $\mathrm{SL}(2, \mathbb{R})$. The conference aimed at elucidating the outstanding problems in the theory of real groups as well as explaining what tools have recently become available to solve them. Since the target audience consisted of graduate student participants, the lectures included a strong instructional component.

The papers in this volume, we hope, fulfill the conference aims, capturing the scope of the theory and the important work which remains. James Arthur poses a list of open questions primarily motivated by considerations of the trace formula, and many of the other contributions in the volume relate directly or indirectly to the problems he considers. Jean-Pierre Labesse gives a very accessible introduction to the theory of endoscopy including many new simplifications in exposition. Diana Shelstad discusses the transfer factors associated to real reductive groups, a major component of Langlands' proposals regarding the application of the trace formula, notably streamlining some of her earlier work. Werner Hoffman deals with weighted orbital integral, objects which are envisioned to feature prominently in the theory beyond endoscopy. Jeffrey Adams discusses `atlas`, a powerful interactive computer program for exploration of the structure and representations of real reductive groups. Bill Casselman's short article is devoted to a theoretical question raised in the implementation of `atlas`. A paper by Dan Barbasch, Dan Ciubotaru, and Alessandra Pantano surveys recent work on the question of unitarizability for representations of both real and p -adic groups, emphasizing the similarities of the two cases and uncovering various sources of functorialities for unitary representations.

On matters more personal, the conference celebrated the birthdays of Dragan Milićić and Bill Casselman. It is only fitting that their influence and energy can be felt on many of the pages which follow.

Finally, it is a pleasure to thank the AMS, especially Wayne Drady and Christine Thivierge, for help both with organizing the conference itself and with the production of this volume.

James Arthur
Wilfried Schmid
Peter E. Trapa

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The representation theory of real reductive groups is still incomplete, in spite of much progress made thus far. The papers in this volume were presented at The AMS-IMS-SIAM Joint Summer Research Conference "Representation Theory of Real Reductive Lie Groups" held in Snowbird, Utah in June 2006, with the aim of elucidating the problems that remain, as well as explaining what tools have recently become available to solve them. They represent a significant improvement in the exposition of some of the most important (and often least accessible) aspects of the literature.

This volume will be of interest to graduate students working in the harmonic analysis and representation theory of Lie groups. It will also appeal to experts working in closely related fields.

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