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Algebraic Methods in Statistics and Probability II

AMS Special Session
Algebraic Methods in Statistics and Probability
March 27–29, 2009
University of Illinois at Urbana-Champaign
Champaign, IL

Marlos A. G. Viana
Henry P. Wynn
Editors



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Special Session on Algebraic Methods in Statistics and Probability, University of Illinois Urbana-Champaign, Noyes Lab, March 28, 2009.

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Preface

Nearly a decade after the publication of (the “South Bend”) CONM Vol.287 *Algebraic Methods in Statistics and Probability*, the present volume may be characterized by the gratifying inclusion of recent material from PhD dissertations and the participation of several colleagues who envisioned the new directions in the field during the past decade. The editors were motivated by this support and trust that the present “Urbana Volume” will have a lasting impact on the field.

The volume demonstrates the consolidation of many of the areas that have developed and acquired their own character during that period, such as algebraic statistics and other applications of computational commutative algebra and deeper aspects of graphical models. These areas have their foundations in computational, algebraic and convex geometry, tropical geometry, algebraic topology, group theory, and harmonic analysis. The ideas reach back to the works of Sir Ronald A. Fisher [1–3], Samuel S. Wilks [4], Alan T. James [5,6], T.W. Anderson [7], Henry B. Mann [8], A.S. McLaren [9], Edward J. Hannan [10], and Gérard Letac [11] among others.

In statistics, the range of topics in the present volume includes new results and applications in cubic regression models for mixture experiments, multidimensional Fourier regression experiments, polynomial characterizations of weakly invariant designs, toric and mixture models for the diagonal-effect in two-way contingency tables, topological methods for multivariate statistics, structural results for the Dirichlet distributions, inequalities for partial regression coefficients, graphical models for binary random variables, conditional independence and its relation to sub-determinants covariance matrices, connectivity of binary tables (via Markov bases), kernel smoothing methods for partially ranked data, combination of graphical models, discrete Fourier transforms and (generic-field) regular designs, Fourier analysis over the dihedral groups, properties of square non-symmetric matrices, and Wishart distributions over symmetric cones.

In probability, the volume includes new results related to discrete-time semi-Markov processes, weak convergence of convolution products in semigroups (of stochastic matrices), Markov bases for directed random graph models, functional analysis in Hardy spaces, and to the Hewitt-Savage zero-one law.

The volume points to several links between many of the fields that may lead to further research. For example, experimental design in the context of contingency tables can be formulated as the study of tables with prohibited cells, so that Markov bases for models over such designs can be better developed; the study of optimal experimental designs might benefit from exploring their symmetries in relation to algebraic arguments and formal invariant theory; computational algebra in main-stream probability theory is ripe for more development and we should be particularly interested when there are applications in statistics. The foundations

in areas like semi-group theory in Markov chains and algebraic combinatorics for counting special configurations using generating function techniques, together with asymptotic theory, may prove to be fruitful leads to statistical applications.

The editors are grateful to all who assisted us during the refereeing process, including Carlos F. Brito, Jean-François Burnol, Yuguo Chen, Jan Draisma, Hamid R. Eghbalian, Russell Lyons, Hélène Massam, František Matúš, Jason Morton, Victor Patrangenaru, Rafael Pérez Ocón, Bhamidi V. Rao, Alberto Roverato, Stephen D. Smith, Nicolas Tserpes, Joseph Verducci, Ruriko Yoshida, and Josephine Yu.

We also acknowledge the support of the series managing editor Dennis DeTurck and the Associate Editor for Proceedings Christine M. Thivierge for her continued guidance during all the phases of the project. Our appreciation extends to Susan J. Friedlander for facilitating our organizing of the special session presented at the Urbana meeting.

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