

CONTEMPORARY MATHEMATICS

520

Algorithmic Probability and Combinatorics

AMS Special Sessions on
Algorithmic Probability and Combinatorics

October 5–6, 2007

DePaul University

Chicago, Illinois

October 4–5, 2008

University of British Columbia

Vancouver, BC, Canada

Manuel E. Lladser
Robert S. Maier
Marni Mishna
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Preface

This volume contains refereed articles by speakers in the AMS Special Sessions on Algorithmic Probability and Combinatorics, held on October 5–6, 2007 at DePaul University in Chicago, IL, and on October 4–5, 2008 at the University of British Columbia in Vancouver, BC. The articles cover a wide range of topics in analytic combinatorics and in the study, both analytic and computational, of combinatorial probabilistic models. The authors include pure mathematicians, applied mathematicians, and computational physicists. A few of the articles have an expository flavor, with extensive bibliographies, but original research predominates.

This is the first volume that the AMS has published in this interdisciplinary area. Our hope is that these articles give an accurate picture of its variety and vitality, and its ties to other areas of mathematics. These areas include asymptotic analysis, algebraic geometry, special functions, the analysis of algorithms, statistical mechanics, stochastic simulation, and importance sampling.

As co-organizers and co-editors, we thank all participants, contributors, and referees. We are grateful to the American Mathematical Society for assistance in organizing the special sessions, and in the publication of this volume. We especially thank Christine Thivierge of the AMS staff, for her efficient support in the latter.

Manuel E. Lladser
Robert S. Maier
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This volume contains the proceedings of the AMS Special Sessions on Algorithmic Probability and Combinatorics held at DePaul University on October 5–6, 2007 and at the University of British Columbia on October 4–5, 2008.

This volume collects cutting-edge research and expository on algorithmic probability and combinatorics. It includes contributions by well-established experts and younger researchers who use generating functions, algebraic and probabilistic methods as well as asymptotic analysis on a daily basis. Walks in the quarter-plane and random walks (quantum, rotor and self-avoiding), permutation tableaux, and random permutations are considered. In addition, articles in the volume present a variety of saddle-point and geometric methods for the asymptotic analysis of the coefficients of single- and multi-variable generating functions associated with combinatorial objects and discrete random structures. The volume should appeal to pure and applied mathematicians, as well as mathematical physicists; in particular, anyone interested in computational aspects of probability, combinatorics and enumeration. Furthermore, the expository or partly expository papers included in this volume should serve as an entry point to this literature not only to experts in other areas, but also to graduate students.

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