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Aspects of Operator Algebras and Applications

UIMP-RSME Lluís A. Santaló Summer School
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Santander, Spain
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Pere Ara
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Preface

This volume contains the expository articles for the lectures given at the "UIMP-RSME *Lluís Santaló* Summer School in Mathematics 2008" with title "Aspects of Operator Algebras and Applications". The goal of the Summer School was to present an advanced introduction to some of the current topics in operator algebras and its applications to mathematical physics. The lectures were directed to young post–docs, advanced doctoral students and researchers interested in this interdisciplinary subject. The Summer School was held at the *Palacio de la Magdalena* in Santander (Spain) on July 21–25, 2008, as part of the activities of the Universidad Internacional Menéndez Pelayo (UIMP).

In recent years, the theory of operator algebras has received a strong impetus in Spain. This is in part due to the celebration of the international event "Barcelona Conference on C*-Algebras and their Invariants" in June 2007, and also the "*Lluís Santaló* Summer School 2008". These events already have a follow-up, in the form of a school and workshop "Topics in Operator Algebras and some Applications" planned for September 2010 in Madrid. This will be a joint activity of the Departments of Mathematics of the *Universidad Carlos III de Madrid* and the *Universitat Autònoma de Barcelona*, as well as the Department of Analysis of the *Universidad Complutense de Madrid*. In addition, there will be an intensive research program on "The Cuntz semigroup and the Classification of C*-algebras", to be held in the *Centre de Recerca Matemàtica* (CRM, Barcelona) in the first semester of 2011. For the latter, a workshop, an advanced course on C*-dynamics and an international conference are also planned. The editors of this book have been actively engaged in the organization of all these events, that undoubtedly confirm Spain as an important emerging country for this area.

The lectures of the *Lluís Santaló* Summer School dealt with different aspects of the theory of operator algebras, ranging from C*-algebras and the Cuntz semigroup to von Neumann algebras, modular theory and applications to mathematical physics. Also many of the known interconnections between the theories of C*-algebras and von Neumann algebras were discussed in the course. These topics were divided into three parts and have been the basis of the articles that constitute the present volume. We broadly outline their contents below.

- Part A: K-Theory and Classification of C*-algebras.
  The research on classification of C*-algebras using the Cuntz semigroup as a main tool is one of the most recent trends in the theory. The theory received a major boost with the publication of a paper by Coward, Elliott and Ivanescu, "The Cuntz semigroup as an invariant for C*-algebras", *J. Reine Angew. Math.* **623** (2008), 161–193. The article of Ara, Perera and Toms in this book presents the main results of the above paper in

a comprehensive way, and provides a fast and direct introduction to the Cuntz semigroup and its role in the classification program.

- Part B: Modular Theory for von Neumann Algebras and Applications to Quantum Field Theory.
  Two articles of this volume deal with this subject. The article by Fernando Lledó includes a short introduction to Modular Theory for von Neumann algebras with a cyclic and separating vector. The theory is illustrated by many examples. In particular, the modular objects are constructed explicitly for a discrete crossed product and, also, for the algebra of the canonical anti-commutation relations (CAR-algebra) in an irreducible representation. The latter results are also applied in the context of fermionic quantum fields.

  The first part of the article by Daniele Guido introduces the axioms of quantum fields and nets of von Neumann algebras indexed by regions in spacetime. The author also presents the free scalar quantum field as the standard example of a bosonic field and discusses classical results (like, e.g., the Bisognano-Wichmann theorem) that show the importance of Modular Theory in this area. In the second part of the article the author addresses recent developments of the subject that involve the modular group. Among other things, notions like modular covariance and modular localization are discussed in detail.
- Part C: Amenability, Hyperbolic Groups and Operator algebras.
  The interplay between C*- and von Neumann algebra theory has a long history with each theory contributing important results to the other. Perhaps the most fruitful context in which to view these interactions is that of amenability. The article by Nate Brown outlines some of the highlights, starting with classical results about amenable groups and ending with recent spectacular work of Ozawa and Popa related to hyperbolic groups.

Finally, we have gathered in an Appendix the basic material on C*-algebras and von Neumann algebras needed to read this book. For the proofs and additional information, we have given precise references to some of the standard textbooks on this flourishing subject.

The editors wish to thank the *Real Sociedad Matemática Española* (RSME) for sponsoring this event. It is a pleasure to thank also the patronage of the UIMP, which has a long tradition in all areas of knowledge, particularly in mathematics. Their help in the organization and the lodging experience in the Palacio de la Magdalena will be remembered in years to come. We also thank the *Comissionat per Universitats i Recerca de la Generalitat de Catalunya* for financial support. We are deeply grateful to Nate Brown, Daniele Guido and Andrew Toms for the great quality of their lectures and also for having contributed to this volume. Last, but certainly not least, we also thank the attendants of this edition of the *Lluís Santaló* Summer School for their interest and dedication. Without them, this meeting would have not been possible.

Pere Ara, Fernando Lledó and Francesc Perera

This volume contains survey papers on the theory of operator algebras based on lectures given at the "Lluís Santaló" Summer School of the Real Sociedad Matemática Española, held in July 2008 at the Universidad Internacional Menéndez Pelayo, in Santander (Spain).

Topics in this volume cover current fundamental aspects of the theory of operator algebras, which have important applications such as:

- K-Theory, the Cuntz semigroup, and Classification for C^*-algebras
- Modular Theory for von Neumann algebras and applications to Quantum Field Theory
- Amenability, Hyperbolic Groups, and Operator Algebras.

The theory of operator algebras, introduced in the thirties by J. von Neumann and F. J. Murray, was developed in close relationship with fundamental aspects of functional analysis, ergodic theory, harmonic analysis, and quantum physics. More recently, this field has shown many other fruitful interrelations with several areas of mathematics and mathematical physics.

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