CONTEMPORARY MATHEMATICS

540

Nonlinear Elliptic Partial Differential Equations

Workshop in Celebration of Jean-Pierre Gossez's 65th Birthday
September 2–4, 2009
Université Libre de Bruxelles, Belgium

Denis Bonheure
Mabel Cuesta
Enrique J. Lami Dozo
Peter Takáč
Jean Van Schaftingen
Michel Willem
Editors

American Mathematical Society

http://dx.doi.org/10.1090/conm/540

Nonlinear Elliptic Partial Differential Equations

CONTEMPORARY MATHEMATICS

540

Nonlinear Elliptic Partial Differential Equations

Workshop in Celebration of Jean-Pierre Gossez's 65th Birthday
September 2–4, 2009
Université Libre de Bruxelles, Belgium

Denis Bonheure
Mabel Cuesta
Enrique J. Lami Dozo
Peter Takáč
Jean Van Schaftingen
Michel Willem
Editors

American Mathematical Society
Providence, Rhode Island

Editorial Board

Dennis DeTurck, managing editor

George Andrews Abel Klein Martin J. Strauss

2000 *Mathematics Subject Classification.* Primary 35B50, 35J15, 35J20, 35J25, 35J60, 35J70, 35P30, 46E35, 49Q20, 58J05.

Library of Congress Cataloging-in-Publication Data

Workshop in Nonlinear Elliptic Partial Differential Equations (2009 : Université Libre de Bruxelles)

Nonlinear elliptic partial differential equations : Workshop in Nonlinear Elliptic Partial Differential Equations in celebration of Jean-Pierre Gossez's 65th birthday, September 2–4, 2009, Université Libre de Bruxelles, Belgium / Denis Bonheure . . . [et al.], editors.

p. cm. — (Contemporary mathematics ; v. 540)

Includes bibliographical references and index.

ISBN 978-0-8218-4907-1 (alk. paper)

1. Differential equations, Elliptic–Congresses. 2. Differential equations, Nonlinear–Congresses. I. Gossez, J.-P., 1943– II. Bonheure, Denis. III. Title.

QA377.W675 2009
515′.3533—dc22

2010049092

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to `reprint-permission@ams.org`.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2011 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights
except those granted to the United States Government.
Copyright of individual articles may revert to the public domain 28 years
after publication. Contact the AMS for copyright status of individual articles.
Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 16 15 14 13 12 11

Contents

Preface vii

Jean-Pierre Gossez's short bio ix

List of Participants xi

Partial differential equations also have principles : maximum and antimaximum
Jean Mawhin 1

On the Fučík Spectrum for equations with symmetries
Bernhard Ruf 15

Variations on the p-Laplacian
Bernd Kawohl 35

Extremal functions in Poincaré–Sobolev inequalities for functions of bounded variation
Vincent Bouchez and Jean Van Schaftingen 47

An elementary proof of an inequality of Maz'ya involving L^1 vector fields
Pierre Bousquet and Petru Mironescu 59

Homoclinic type solutions for a class of differential equations with periodic coefficients
David G. Costa and Chengyue Li 65

Quasilinear and singular systems: the cooperative case
Jacques Giacomoni, Jesús Hernández,
and Abdelkrim Moussaoui 79

Manifolds of critical points in a quasilinear model for phase transitions
Pavel Drábek, Raúl F. Manásevich, and Peter Takáč 95

Weighted asymmetric problems for an indefinite elliptic operator
Liamidi Leadi and Humberto Ramos Quoirin 135

Multiple non-trivial solutions of the Dirichlet problem for the prescribed mean curvature equation
Franco Obersnel and Pierpaolo Omari 165

Limits as $p(x) \to \infty$ of $p(x)$-harmonic functions with non-homogeneous Neumann boundary conditions
Mayte Perez-LLanos and Julio D. Rossi 187

Bifurcation and decay of solutions for a class of elliptic equations on $\mathbb{R}^N$
C. A. Stuart 203

Existence of nodal solutions for some nonlinear elliptic problems
Sébastien de Valeriola and Michel Willem 231

Admissible $Q-$curvatures under isometries for the conformal GJMS operators
Frédéric Robert 241

Preface

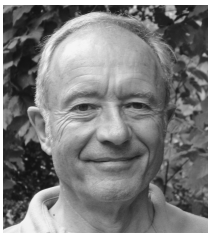
A workshop celebrating Jean-Pierre Gossez's 65th anniversary was organized on September 2, 3 and 4, 2009. This meeting was attended by around 80 young and senior researchers. As usual in such occasions, it was mainly a good opportunity to gather old friends and for the youngest to make new contacts. The atmosphere was warm and very friendly, fruitful for an excellent workshop with beautiful lectures and many interesting and inspiring scientific discussions. The 16 lecturers were Henri Berestycki, Djairo G. de Figueiredo, François de Thélin, Pavel Drábek, Jesús Hernández, Bernd Kawohl, Pierre-Louis Lions, Jean Mawhin, Petru Mironescu, Mark Peletier, Frédéric Robert, Julio Daniel Rossi, Bernhard Ruf, Michael Struwe, Charles Stuart and Juan Luis Vázquez.

We would like to express our gratitude to the participants and the lecturers, to the persons who helped in the organization of the meeting and to the following institutions that supported the conference : Fonds de la recherche scientifique (F.R.S.-FNRS), Gouvernement de la Région de Bruxelles-Capitale, Académie Universitaire Wallonie-Bruxelles, European Science Foundation, International Mathematical Union, International Solvay Institutes Brussels, Faculté SOCO, Faculté des Sciences and Solvay Brussels School of Economics and Management.

We offer our thanks to the contributors to this volume and to the AMS for giving us the possibility to publish it in the Contemporary Mathematics Series. The efficient help of Christine Thivierge was greatly appreciated.

The Editors

Jean-Pierre Gossez's short bio



Jean-Pierre Gossez was born in Mons (Belgium) the 28th of November 1943. He graduated at the Université Libre de Bruxelles (ULB) in 1965 in Mathematical Sciences, then spent three years in Paris under the supervision of Jacques-Louis Lions and took his Ph.D. at the ULB in 1969 under the direction of Théophile Lepage. Later he was successively postdoctoral researcher at the Fonds National de la Recherche Scientifique, research associate at the University of Chicago and chargé de cours at the ULB. Since 1979, he has been Professeur ordinaire at the ULB where he taught analysis and general mathematics to undergraduate students and the theory of partial differential equations at the graduate level. He is Professeur émérite at the ULB since October 1st, 2009.

His research is mainly concerned with the theory of elliptic PDEs. At first, he studied monotone operators in Orlicz-Sobolev spaces and their applications to nonlinear boundary value problems. Then he took an active interest in nonlinear resonance problems. He considered non resonance conditions at various eigenvalues, problems with asymmetric nonlinearities, the Fučik spectrum,... for the Laplacian and the p-Laplacian. Fifteen years ago, he got interested in the antimaximum principle, first for the p-Laplacian, then for semilinear and quasilinear problems with weights. His research on the antimaximum principle lead him to consider some minimax formulas for the principal eigenvalues, and more recently he investigated non selfadjoint elliptic problems.

He has been involved in cooperation programmes with North-Africa and Latin-America. He is member of the Commission for Development and Exchange and of the committee "Developping Countries Strategic Group" of the International Mathematical Union.

List of Participants

(1) Margarita Arias, Universidad de Granada
marias@ugr.es
(2) Tapio Behrndt, University of Oxford
behrndt@maths.ox.ac.uk
(3) Nitsan Ben-Gal, Brown University
bengal@dam.brown.edu
(4) Henri Berestycki, École des Hautes Études en Sciences Sociales
hb@ehess.fr
(5) Adam Besenye, Eötvös Lorand University
badam@cs.elte.hu
(6) Denis Bonheure, Université Libre de Bruxelles
dbonheur@ulb.ac.be
(7) Mohammed Bouchekif, University of Tlemcen
m_bouchekif@yahoo.fr
(8) Vincent Bouchez, Université catholique de Louvain
vincent.bouchez@uclouvain.be
(9) Juan Campos, Universidad de Granada
campos@ugr.es
(10) Mathieu Carette, Université Libre de Bruxelles
mcarette@ulb.ac.be
(11) Yuanji Cheng, Malmö University
yuanji.cheng@mah.se
(12) Jean Cortissoz, Universidad de Los Andes
jean.cortissoz@gmail.com
(13) Mabel Cuesta, Université du Littoral Côte d'Opale
cuesta@lmpa.univ-littoral.fr
(14) Djairo Guedes de Figueiredo, Universidade Estadual de Campinas
djairo@ime.unicamp.br
(15) Pieter de Groen, Vrije Universiteit Brussel
pdegroen@vub.ac.be
(16) Xavier De Scheemaekere, Université Libre de Bruxelles
xdeschee@ulb.ac.be
(17) Franois de Thélin, Université Paul Sabatier
dethelin@mip.ups-tlse.fr
(18) Sbastien de Valeriola, Université catholique de Louvain
sebastien.devaleriola@uclouvain.be
(19) Ann Derlet, Université Libre de Bruxelles
aderlet@ulb.ac.be

(20) Bertrand Desmons, Université de Mons
bertrand.desmons@umons.ac.be
(21) Jonathan Di Cosmo, Université catholique de Louvain
jonathan.dicosmo@uclouvain.be
(22) Joo Marcos do Ó, Universidade Federal da Paraba
jmbo@mat.ufpb.br
(23) Pavel Drábek, University of West Bohemia
pdrabek@kma.zcu.cz
(24) Louis Dupaigne, Université Picardie Jules Vernes
louis.dupaigne@u-picardie.fr
(25) Gilles Evequoz, Universität Karlsruhe
gilles.evequoz@math.uni-karlsruhe.de
(26) Paul Godin, Université Libre de Bruxelles
pgodin@ulb.ac.be
(27) Tómas Fernando Godoy, Universidad Nacional de Córdoba
godoy@mate.uncor.edu
(28) Kazimierz Goebel, Maria Curie-Sklodowska University
goebel@hektor.umcs.lublin.pl
(29) Jean-Pierre Gossez, Université Libre de Bruxelles
gossez@ulb.ac.be
(30) Antonio Greco, University of Cagliari
greco@unica.it
(31) Christopher Grumiau, Université de Mons
christopher.grumiau@umons.ac.be
(32) Patrick Habets, Université catholique de Louvain
Patrick.Habets@uclouvain.be
(33) Jesús Hernández, Universidad Autónoma de Madrid
jesus.hernandez@uam.es
(34) Norimichi Hirano, Yokohama National University
hira0918@gmail.com
(35) Gabriela Holubová, University of West Bohemia
gabriela@kma.zcu.cz
(36) Shu-Yu Hsu, National Chung-Cheng University
syhsu@math.edu.tw
(37) Bernd Kawohl, Universität zu Köln
kawohl@math.uni-koeln.de
(38) Piotr Kokocki, Nicolaus Copernicus University
koksi@mat.umk.pl
(39) Kundan Kumar, Technische Universiteit Eindhoven
k.kumar@tue.nl
(40) Sophia Kyritsi, Hellenic Naval Academy
skyrits@math.ntua.gr
(41) El Haj Laamri, Université de Nancy 2
Elhaj.Laamri@iecn.u-nancy.fr
(42) Enrique Lami Dozo, Université Libre de Bruxelles
lamidozo@ulb.ac.be
(43) Liamidi Leadi, Université d'Abomey Calavi
leadiare@yahoo.com

(44) Pierre-Louis Lions, Collège de France
lions@dmi.ens.fr
(45) Ahmed Loulit, Université Libre de Bruxelles
hloulit@ulb.ac.be
(46) Walcaw Marzantowicz, UAM Poznan
marzan@main.amu.edu.pl
(47) Jean Mawhin, Université catholique de Louvain
mawhin@math.ucl.ac.be
(48) Carlo Mercuri, SISSA
mercuri@sissa.it
(49) Petru Mironescu, Université Claude Bernard Lyon 1
mironescu@math.univ-lyon1.fr
(50) Olmpio Miyagaki, Universidade Federal de Vicosa
olimpio@ufv.br
(51) Petr Necesal, University of West Bohemia
pnecesal@kma.zcu.cz
(52) Sofia Paczka, Universidad Nacional de Cordoba
paczka@mate.uncor.edu
(53) Vassilis Papanicolaou, National Technical Univ. Athens
papanico@math.ntua.gr
(54) Enea Parini, Universität zu Köln
eparini@math.uni-koeln.de
(55) Mark Peletier, Technische Universiteit Eindhoven
m.a.peletier@tue.nl
(56) Augusto Ponce, Université catholique de Louvain
Augusto.Ponce@uclouvain.be
(57) Alessandra Ragusa, University of Catania
maragusa@dmi.unict.it
(58) Humberto Ramos Quoirin, Université Libre de Bruxelles
hramosqu@ulb.ac.be
(59) Frédéric Robert, Université de Nice Sophia-Antipolis
frobert@math.unice.fr
(60) Julio Daniel Rossi, Universidad de Buenos Aires
jrossi@dm.uba.ar
(61) Bernhard Ruf, Universit degli Studi di Milano
ruf@mat.unimi.it
(62) Krzysztof Rykaczewski, Nicolaus Copernicus University
krykaczewski@gmail.com
(63) Abderrahmane Senoussaoui, Université d'Oran Es-Senia
senoussaoui_abdou@yahoo.fr
(64) László Simon, Eötvös Lorand University
simonl@ludens.elte.hu
(65) Peter Simon, Eötvös Lorand University
simonp@cs.elte.hu
(66) Boyan Sirakov, École des Hautes Études en Sciences Sociales
bsirakov@yahoo.com
(67) Michael Struwe, ETH Zrich
michael.struwe@math.ethz.ch

(68) Charles Stuart, École Polytechnique Fédérale de Lausanne
charles.stuart@epfl.ch
(69) Peter Takác, Universität Rostock
peter.takac@uni-rostock.de
(70) Thuan Tran Ba, Université catholique de Louvain
thuan.tranba@uclouvain.be
(71) Christophe Troestler, Université de Mons
Christophe.Troestler@umh.ac.be
(72) Pedro Ubilla, Universidad de Santiago de Chile
pubilla@fermat.usach.cl
(73) Jean Van Schaftingen, Université catholique de Louvain
jean.vanschaftingen@uclouvain.be
(74) André Vanderbauwhede, Ghent University
avdb@cage.ugent.be
(75) Juan Louis Vázquez, Universidad Autónoma de Madrid
juanluis.vazquez@uam.es
(76) Michel Willem, Université catholique de Louvain
willem@math.ucl.ac.be
(77) Theresia Yiallourou, University of Cyprus
me06gt1@ucy.ac.cy
(78) Nikolaus Zographopoulos, Technical University of Crete
nzogr@science.tuc.gr

Titles in This Series

540 **Denis Bonheure, Mabel Cuesta, Enrique J. Lami Dozo, Peter Takáč, Jean Van Schaftingen, and Michel Willem, Editors,** Nonlinear Elliptic Partial Differential Equations, 2011

539 **Kurusch Ebrahimi-Fard, Matilde Marcolli, and Walter D. van Suijlekom, Editors,** Combinatorics and Physics, 2011

538 **José Ignacio Cogolludo-Agustín and Eriko Hironaka, Editors,** Topology of algebraic varieties and singularities, 2011

537 **César Polcino Milies, Editor,** Groups, algebras and applications, 2011

536 **Kazem Mahdavi, Debbie Koslover, and Leonard L. Brown III, Editors,** Cross disciplinary advances in quantum computing, 2011

535 **Maxim Braverman, Leonid Friedlander, Thomas Kappeler, Peter Kuchment, Peter Topalov, and Jonathan Weitsman, Editors,** Spectral theory and geometric analysis, 2011

534 **Pere Ara, Fernando Lledó, and Francesc Perera, Editors,** Aspects of operator algebras and applications, 2011

533 **L. Babinkostova, A. E. Caicedo, S. Geschke, and M. Scheepers, Editors,** Set theory and its applications, 2011

532 **Sergiy Kolyada, Yuri Manin, Martin Möller, Pieter Moree, and Thomas Ward, Editors,** Dynamical numbers: Interplay between dynamical systems and number theory, 2010

531 **Richard A. Brualdi, Samad Hedayat, Hadi Kharaghani, Gholamreza B. Khosrovshahi, and Shahriar Shahriari, Editors,** Combinatorics and graphs, 2010

530 **Vitaly Bergelson, Andreas Blass, Mauro Di Nasso, and Renling Jin, Editors,** Ultrafilters across Mathematics, 2010

529 **Robert Sims and Daniel Ueltschi, Editors,** Entropy and the Quantum, 2010

528 **Alberto Farina and Enrico Valdinoci, Editors,** Symmetry for Elliptic PDEs, 2010

527 **Ricardo Castaño-Bernard, Yan Soibelman, and Ilia Zharkov, Editors,** Mirror symmetry and tropical geometry, 2010

526 **Helge Holden and Kenneth H. Karlsen, Editors,** Nonlinear partial differential equations and hyperbolic wave phenomena, 2010

525 **Manuel D. Contreras and Santiago Díaz-Madrigal, Editors,** Five lectures in complex analysis, 2010

524 **Mark L. Lewis, Gabriel Navarro, Donald S. Passman, and Thomas R. Wolf, Editors,** Character theory of finite groups, 2010

523 **Aiden A. Bruen and David L. Wehlau, Editors,** Error-correcting codes, finite geometries and cryptography, 2010

522 **Oscar García-Prada, Peter E. Newstead, Luis Álverez-Cónsul, Indranil Biswas, Steven B. Bradlow, and Tomás L. Gómez, Editors,** Vector bundles and complex geometry, 2010

521 **David Kohel and Robert Rolland, Editors,** Arithmetic, geometry, cryptography and coding theory 2009, 2010

520 **Manuel E. Lladser, Robert S. Maier, Marni Mishna, and Andrew Rechnitzer, Editors,** Algorithmic probability and combinatorics, 2010

519 **Yves Félix, Gregory Lupton, and Samuel B. Smith, Editors,** Homotopy theory of function spaces and related topics, 2010

518 **Gary McGuire, Gary L. Mullen, Daniel Panario, and Igor E. Shparlinski, Editors,** Finite fields: Theory and applications, 2010

517 **Tewodros Amdeberhan, Luis A. Medina, and Victor H. Moll, Editors,** Gems in experimental mathematics, 2010

516 **Marlos A.G. Viana and Henry P. Wynn, Editors,** Algebraic methods in statistics and probability II, 2010

515 **Santiago Carrillo Menéndez and José Luis Fernández Pérez, Editors,** Mathematics in finance, 2010

514 **Arie Leizarowitz, Boris S. Mordukhovich, Itai Shafrir, and Alexander J. Zaslavski, Editors,** Nonlinear analysis and optimization II, 2010

513 **Arie Leizarowitz, Boris S. Mordukhovich, Itai Shafrir, and Alexander J. Zaslavski, Editors,** Nonlinear analysis and optimization I, 2010

512 **Albert Fathi, Yong-Geun Oh, and Claude Viterbo, Editors,** Symplectic topology and measure preserving dynamical systems, 2010

511 **Luise-Charlotte Kappe, Arturo Magidin, and Robert Fitzgerald Morse, Editors,** Computational group theory and the theory of groups, II, 2010

510 **Mario Bonk, Jane Gilman, Howard Masur, Yair Minsky, and Michael Wolf, Editors,** In the Tradition of Ahlfors-Bers, V, 2010

509 **Primitivo B. Acosta-Humánez and Francisco Marcellán, Editors,** Differential algebra, complex analysis and orthogonal polynomials, 2010

508 **Martin Berz and Khodr Shamseddine, Editors,** Advances in p-Adic and non-archimedean analysis, 2010

507 **Jorge Arvesú, Francisco Marcellán, and Andrei Martínez-Finkelshtein, Editors,** Recent trends in orthogonal polynomials and approximation theory, 2010

506 **Yun Gao, Naihuan Jing, Michael Lau, and Kailash C. Misra, Editors,** Quantum affine algebras, extended affine Lie algebras, and their applications, 2010

505 **Patricio Cifuentes, José García-Cuerva, Gustavo Garrigós, Eugenio Hernández, José María Martell, Javier Parcet, Alberto Ruiz, Fernándo Soria, José Luis Torrea, and Ana Vargas, Editors,** Harmonic analysis and partial differential equations, 2010

504 **Christian Ausoni, Kathryn Hess, and Jérôme Scherer, Editors,** Alpine perspectives on algebraic topology, 2009

503 **Marcel de Jeu, Sergei Silvestrov, Christian Skau, and Jun Tomiyama, Editors,** Operator structures and dynamical systems, 2009

502 **Viviana Ene and Ezra Miller, Editors,** Combinatorial Aspects of Commutative Algebra, 2009

501 **Karel Dekimpe, Paul Igodt, and Alain Valette, Editors,** Discrete groups and geometric structures, 2009

500 **Philippe Briet, François Germinet, and Georgi Raikov, Editors,** Spectral and scattering theory for quantum magnetic systems, 2009

499 **Antonio Giambruno, César Polcino Milies, and Sudarshan K. Sehgal, Editors,** Groups, rings and group rings, 2009

498 **Nicolau C. Saldanha, Lawrence Conlon, Rémi Langevin, Takashi Tsuboi, and Pawel Walczak, Editors,** Foliations, geometry and topology, 2009

497 **Maarten Bergvelt, Gaywalee Yamskulna, and Wenhua Zhao, Editors,** Vertex operator algebras and related areas, 2009

496 **Daniel J. Bates, GianMario Besana, Sandra Di Rocco, and Charles W. Wampler, Editors,** Interactions of classical and numerical algebraic geometry, 2009

495 **G. L. Litvinov and S. N. Sergeev, Editors,** Tropical and idempotent mathematics, 2009

494 **Habib Ammari and Hyeonbae Kang, Editors,** Imaging microstructures: Mathematical and computational challenges, 2009

493 **Ricardo Baeza, Wai Kiu Chan, Detlev W. Hoffmann, and Rainer Schulze-Pillot, Editors,** Quadratic Forms—Algebra, Arithmetic, and Geometry, 2009

492 **Fernando Giráldez and Miguel A. Herrero, Editors,** Mathematics, Developmental Biology and Tumour Growth, 2009

491 **Carolyn S. Gordon, Juan Tirao, Jorge A. Vargas, and Joseph A. Wolf, Editors,** New developments in Lie theory and geometry, 2009

For a complete list of titles in this series, visit the AMS Bookstore at **www.ams.org/bookstore/**.

This volume contains papers on semi-linear and quasi-linear elliptic equations from the workshop on Nonlinear Elliptic Partial Differential Equations, in honor of Jean-Pierre Gossez's 65th birthday, held September 2–4, 2009 at the Université Libre de Bruxelles, Belgium.

The workshop reflected Gossez's contributions in nonlinear elliptic PDEs and provided an opening to new directions in this very active research area. Presentations covered recent progress in Gossez's favorite topics, namely various problems related to the p-Laplacian operator, the antimaximum principle, the Fučík Spectrum, and other related subjects. This volume will be of principle interest to researchers in nonlinear analysis, especially in partial differential equations of elliptic type.

ISBN 978-0-8218-4907-1

9 780821 849071

CONM/540

AMS *on the* Web
www.ams.org