CONTEMPORARY MATHEMATICS

546

Noncommutative Geometry and Global Analysis

Conference in Honor of Henri Moscovici
June 29–July 4, 2009
Bonn, Germany

Alain Connes
Alexander Gorokhovsky
Matthias Lesch
Markus Pflaum
Bahram Rangipour
Editors

American Mathematical Society

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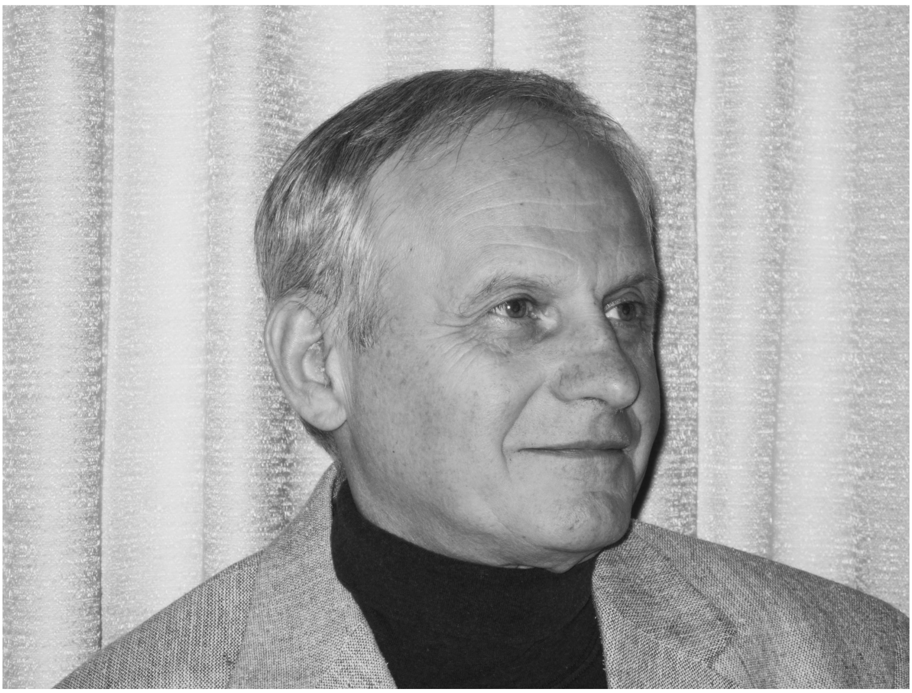

H. Moscovici

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Introduction

This volume represents the proceedings of the conference in honor of Henri Moscovici held in Bonn, June 29 – July 4, 2009. Moscovici has made fundamental contributions to Noncommutative Geometry, Global Analysis and Representation Theory. Especially, his 30-year long collaboration with A. Connes has led to a number of foundational results. They obtained an L^2-index theorem for homogeneous spaces of general Lie groups, generalizing the Atiyah-Schmid theorem for semisimple Lie groups and a higher index theorem for multiply connected manifolds. The latter played a key role in their proof of the Novikov conjecture for word-hyperbolic groups. In the course of their work on the transverse geometry of foliations they discovered the local index formula for spectral triples. The calculations based on the latter formula provided the impetus for the development of the cyclic cohomology theory of Hopf algebras.

In the words of Alain Connes: "I have always known Henri as a prince who escaped from the dark days of the communist era in Romania. With his Mediterranean charm and his intense intelligence, so often foresighted, but never taking himself too seriously, with his inimitable wit, and his constant regard for others he certainly stands out among mathematicians as a great and lovable exception."

The present volume, which includes articles by leading experts in the fields mentioned above, provides a panoramic view of the interactions of noncommutative geometry with a variety of areas of mathematics. It contains several surveys as well as high quality research papers. In particular, it focuses on the following themes: geometry, analysis and topology of manifolds and singular spaces, index theory, group representation theory, connections between noncommutative geometry and number theory and arithmetic geometry, Hopf algebras and their cyclic cohomology.

We now give brief summaries of the papers appearing in this volume.

1. D. Barbasch and P. Pandžić **"Dirac cohomology, unipotent representations."**

   In this paper the authors study the problem of classifying unitary representations with Dirac cohomology, focusing on the case when the group G is a complex group viewed as a real group. They conjecture precise conditions under which a unitary representation has nonzero Dirac cohomology and show that it is sufficient.

2. P. Bressler, A. Gorokhovsky, R. Nest and B. Tsygan **"Algebraic index theorem for symplectic deformations of gerbes."**

   Gerbes and twisted differential operators play an increasingly important role in global analysis. In this paper the authors continue their study of the algebraic analogues of the algebra of twisted symbols, namely of the formal deformations of gerbes. They extend the algebraic index theorem of Nest and Tsygan to the deformations of gerbes.

3. J. Brüning, F. W. Kamber and K. Richardson **"Index theory for basic Dirac operators"**

   In this paper the authors consider the transverse Dirac operator on the Riemannian foliation. It has been known for a long time that restricted to the space of holonomy-invariant sections this operator is Fredholm, however there was no formula for the index. Here such a formula is derived, expressing the index in terms of the integrals of characteristic forms and η-invariants of certain operators on the strata of the leaf closure space, provided by Molino's theory.

4. A. Connes **"The Witt construction in characteristic one and quantization."**

   This paper develops the analogue of the Witt construction in characteristic one. The author constructs a functor from pairs (R, ρ) of a perfect semi-ring R of characteristic one and an element $\rho > 1$ of R to real Banach algebras. Remarkably the entropy function occurs uniquely as the analogue of the Teichmüller polynomials in characteristic one. This construction is then applied to the semi-field which plays a central role in idempotent analysis and tropical geometry. It gives the inverse process of the "dequantization" and provides a first hint towards an extension of the field of real numbers relevant both in number theory and quantum physics.

5. M. Dubois-Violette and G. Landi **"Lie prealgebras."**

   This paper studies a generalization of Lie algebras based on the theory of non-homogeneous quadratic algebras. It combines a review of existing theory of quadratic homogeneous algebras with a proposal for a similar theory for the non-homogeneous case. It describes in detail examples of the universal enveloping algebras and the Lie-type algebra associated to the 3D calculus on a twisted or quantum $SU(2)$ group.

6. N. Higson **"On the analogy between complex semisimple groups and their Cartan motion groups."**

   This paper provides a detailed exposition of how to make Mackey's analogy between the representations of a complex semisimple Lie group and its Cartan motion group more precise, using representations of Hecke algebras. The author takes the algebraic approach and obtains a Mackey-type bijection between the admissible dual of a complex semisimple group and that of its motion group.

7. A. Kaygun **"A survey on Hopf-cyclic cohomology and Connes-Moscovici characteristic map."**

   This paper reviews developments of Hopf cyclic cohomology and Connes-Moscovici characteristic map. The author covers almost all topics related to the Hopf cyclic cohomology. He starts with the origins of Hopf cyclic cohomology and covers the further developments including variants of the theory and cup products.

8. M. Khalkhali and A. Pourkia **"A super version of Connes-Moscovici Hopf algebra."**

   In this paper the authors construct an analogue of Connes-Moscovici Hopf algebra associated with the supergroup of diffeomorphisms of a superline $\mathbb{R}^{1,1}$.

They show that, similarly to the Connes-Moscovici Hopf algebra, this super Hopf algebra can be realized as a bicrossed product.

9. V. Mathai and S. Wu **"Analytic torsion of $\mathbb{Z}_2$-graded elliptic complexes."**
   This paper provides a construction of the analytic torsion for arbitrary $\mathbb{Z}_2$-graded elliptic complexes. It extends the construction of the analytic torsion for twisted de Rham complex described in the previous work of the authors. As an illustration an analytic torsion of twisted Dolbeault complexes is defined. The authors also discuss applications of their results to topological field theories.

10. B. Monthubert and V. Nistor **"The K-groups and the index theory of certain comparison C^*-algebras."**
    In this paper the authors consider a comparison algebra for a complete Riemannian manifold which is the interior of a manifold with corners. This is an algebra generated by the 0-order operators which are compositions of differential operators with the inverse powers of the Laplacian. Relating this algebra to the algebra of pseudodifferential operators on a suitable groupoid, they perform calculations of the K-theory of this algebra and apply it to the index problems.

11. H. Moriyoshi and P. Piazza **"Relative pairings and the Atiyah-Patodi-Singer index formula for the Godbillon-Vey cocycle."**
    This paper outlines the authors' approach to the extension of the Moriyoshi-Natsume explicit formula for the pairing between the Godbillon-Vey cyclic cohomology class and the K-theory index class of the longitudinal Dirac operator to foliated bundles on the manifolds with boundary.

12. A. Némethi **"Two exact sequences for lattice cohomology."**
    In the earlier work the author introduced lattice cohomology with the goal of providing a combinatorial description of the Heegaard–Floer homology of Ozsváth and Szabó for the links of normal surface singularities. This has been accomplished for several classes of examples but remains a conjecture in general. In the meantime, the lattice cohomology has become an important tool in studying links of singularities in its own right. This paper develops further properties of the lattice cohomology.

13. B. Rangipour **"Cup products in Hopf cyclic cohomology with coefficients in contramodules."**
    This paper gives a refinement of the cup product construction in Hopf cyclic cohomology, defined in the previous work of M. Khalkhali and the author. This construction is particularly useful for study of the dependance of the cup product on the coefficients.

14. M. Wodzicki **"Algebras of p-symbols, noncommutative p-residue, and the Brauer group"**
    This paper introduces the algebra of p-symbols, a characteristic p analogue of the algebra of pseudodifferential symbols. The author shows that these algebras have some remarkable properties and gives a construction of the noncommutative residue for these algebras. Using elementary but subtle means the author obtains deep and interesting results.

15. G. Yu **"Large scale geometry and its applications."**
    The ideas of large scale geometry played an important role in geometry, topology and group theory beginning with the works of Mostow, Margulis and Gromov. Recently there have been a number of important developments in this area. This article surveys these developments and their applications to geometry and topology of manifolds.

Acknowledgments. First, we would like to thank all people and institutions who helped us to organize this conference. We received generous financial and logistical support from the Hausdorff Center for Mathematics in Bonn. Its staff, especially Anke Thiedemann and Laura Siklossy, provided an extensive help with the organization, and Heike Bacher and Kerstin Strehl-Müller handled the registration. The assistants Batu Güneysu, Benjamin Himpel, Carolina Neira Jiménez, and Boris Vertman helped with the lecture room technology.

We would like to warmly thank Arthur Greenspoon for copy-editing the papers for these proceedings, and Dr. John R. Copeland for providing the photograph of H Moscovici. We also express our thanks to Christine Thivierge, Boris Vertman and Sam White for their assistance in preparing this volume.

Alain Connes
Alexander Gorokhovsky
Matthias Lesch
Markus J. Pflaum
Bahram Rangipour

Conference Talks

This section lists all the talks at the conference together with the speakers' abstracts.

Dirac Cohomology and unipotent representations

Dan Barbasch

In this talk we study the problem of classifying unitary representations with Dirac cohomology. We will focus on the case when the group G is a complex reductive group viewed as a real group. It will easily follow that a necessary condition for having nonzero Dirac cohomology is that twice the infinitesimal character is regular and integral. The main conjecture is the following.

Conjecture: Let G be a complex reductive Lie group viewed as a real group, and π be an irreducible unitary representation such that twice the infinitesimal character of π is regular and integral. Then π has nonzero Dirac cohomology if and only if π is cohomologically induced from an *essentially unipotent* representation with nonzero Dirac cohomology. Here by an essentially unipotent representation we mean a unipotent representation tensored with a unitary character. This work is joint with Pavle Pandzic.

Equivariant K-homology

Paul Frank Baum

K-homology is the dual theory to K-theory. There are two points of view on K-homology: BD (Baum-Douglas) and Atiyah-Kasparov. The BD approach defines K-homology via geometric cycles. The resulting theory in a certain sense is simpler and more direct than classical homology. For example, K-homology and K-theory are made into equivariant theories in an utterly immediate and canonical way. For classical (co)homology, there is an ambiguity about what is the "correct" definition of equivariant (co)homology. In the case of twisted K-homology, the cycles of the BD theory are the D-branes of string theory. This talk will give the definition of equivariant BD theory and its extension to a bivariant theory. An application to the BC (Baum-Connes) conjecture will be explained. The above is joint work with N. Higson, H. Oyono-Oyono and T. Schick.

Moduli spaces of vector bundles on non-Kähler Calabi-Yau type 3-folds

Vasile Brinzanescu

We compute the relative Jacobian of a principal elliptic bundle as a coarse moduli space and find out that it is the product of the fiber with the basis. Using the relative Jacobian we adapt the construction of Caldararu to our case obtaining a twisted Fourier-Mukai transform. Using this transform and the spectral cover we prove that the moduli space of rank n, relatively semi-stable vector bundles is corepresented by the relative Douady space of length n and relative dimension 0 subspaces of the relative Jacobian.

The signature operator on Riemannian pseudomanifolds

Jochen Brüning

We consider an oriented Riemannian manifold which can be compactified by adjoining a smooth compact oriented Riemannian manifold, B, of codimension at least two, such that a neighbourhood of the singular stratum is given by a family of metric cones. We show that there is a natural self-adjoint extension for the Dirac operator on smooth compactly supported differential forms with discrete spectrum, and we determine the condition of essential self-adjointness. We describe the boundary conditions analytically and construct a good parametrix which leads to the asymptotic expansion of the associated heat trace. We also give a new proof of the local formula for the L^2-signature.

The lost Riemann-Roch index problem

Alain Connes

I will describe recent results of joint work with C. Consani. We determined the real counting function $N(q), (q \in (1, \infty))$ for the hypothetical "curve" $C = \overline{\operatorname{Spec} \mathbb{Z}}$ over F_1, whose corresponding zeta function is the complete Riemann zeta function. Then, we develop a theory of functorial F_1-schemes which reconciles the previous attempts by C. Soulé and A. Deitmar. Our construction fits with the geometry of monoids of K. Kato, is no longer limited to toric varieties and covers the case of Chevalley groups. Finally we show, using the monoid of adèle classes over an arbitrary global field, how to apply our functorial theory of $\mathfrak{Mo}$-schemes to interpret conceptually the spectral realization of zeros of L-functions. I will end the lecture by a speculation concerning a Riemann-Roch index problem which is so far lost in a translation.

C^*-algebras associated with integral domains

Joachim Cuntz

We associate canonically a C^*-algebra with every (countable) integral domain. We describe different realizations of this algebra which are used to analyze its structure and K-theory.

The Baum-Connes conjecture and parametrization of group representations

Nigel Higson

Associated to any connected Lie group G is its so-called contraction of G to a maximal compact subgroup. This is a smooth family of Lie groups, and a consequence of the Baum-Connes conjecture is that the reduced duals of all the groups in the family are the same, at least at the level of K-theory. A rather surprising development from the last several years is that in key cases the duals are actually the same at the level of sets. I shall report on recent efforts, both algebraic and geometric, to understand this phenomenon better.

Hopf-cyclic cohomology and Connes-Moscovici characteristic map

Atabey Kaygun

In 1998 Alain Connes and Henri Moscovici invented a cohomology theory for Hopf algebras and a characteristic map associated with the cohomology theory in order to solve a specific technical problem in transverse index theory. In the following decade, the cohomology theory they invented developed on its own under the name Hopf-cyclic cohomology. But the history of Hopf-cyclic cohomology and the characteristic map they invented remained intricately linked. In this survey talk, I will give an account of the development of the characteristic map and Hopf-cyclic cohomology.

Holomorphic structures on the quantum projective line

Masoud Khalkhali

In this talk I report on our joint ongoing work with Giovanni Landi and Walter van Suijlekom. We define a notion of holomorphic structure in terms of a bigrading of a suitable differential calculus over the quantum sphere. Realizing the quantum sphere as a principal homogeneous space of the quantum group $SU_q(2)$ plays an important role in our approach. We define a notion of holomorphic vector bundle and endow the canonical line bundles over the quantum sphere with a holomorphic structure. We also define the quantum homogeneous coordinate ring of the projective line $\mathbb{C}P^1_q$ and identify it with the coordinate ring of the quantum plane. Finally I shall formulate an analogue of Connes' theorem, characterizing holomorphic structures on compact oriented surfaces in terms of positive currents, to our noncommutative context. The notion of twisted positive Hochschild cocycles plays an important role here.

Monopoles connections on the quantum projective plane

Giovanni Landi

We present several results on the geometry of the quantum projective plane. They include: explicit generators for the K-theory and the K-homology; a real calculus with a Hodge star operator; anti-selfdual connections on line bundles with explicit computation of the corresponding invariants; quantum invariants via equivariant K-theory and q-indices; and more.

An index theorem in differential K-theory

John Lott

Differential K-theory is a refinement of the usual K-theory of a manifold. Its objects consist of a vector bundle with a Hermitian inner product, a compatible connection and an auxiliary differential form. Given a fiber bundle with a Riemannian structure on its fibers, and a differential K-theory class on the total space, I will define two differential K-theory classes on the base. These can be considered to be topological and analytic indices. The main result is that they are the same. This is joint work with Dan Freed.

Noncommutative geometry and cosmology: a progress report

Matilde Marcolli

I will describe some ongoing work in collaboration with Elena Pierpaoli (Astronomy, USC/Caltech) on cosmological implications of the noncommutative geometry approach to the standard model with neutrino mixing coupled to gravity previously developed in joint work with Chamseddine and Connes. In particular, I will show how applying the renormalization group flow for the standard model with Majorana mass terms for right handed neutrinos to the asymptotic formula for the spectral action one recovers naturally a wide range of theoretical cosmological models of the inflationary epoch in the early universe.

Connes-Chern character and higher eta cocycles

Henri Moscovici

An intriguing avatar of Connes' Chern character in K-homology assumes the form of a higher eta cocycle. After recounting some previous occurrences of these cocycles, in work of Connes and myself, and also of F. Wu and Getzler, we shall explain how such higher eta cochains and their b-trace analogues can be assembled together to produce representations in relative cyclic cohomology for the Connes-Chern character of a Dirac operator on a manifold with boundary. The latter result is joint work with M. Lesch and M. Pflaum.

Dynamical zeta functions and analytic torsion of hyperbolic manifolds

Werner Müller

In this talk we discuss the relation between the Ruelle zeta function and the analytic torsion of a hyperbolic manifold. In particular, we derive an asymptotic formula for the analytic torsion of a hyperbolic 3-manifold with respect to a special sequence of representations of the fundamental group. We apply this formula to study the torsion of the cohomology of arithmetic hyperbolic 3-manifolds.

Lattice (co)homology associated with plumbed 3-manifolds

András Némethi

For any negative definite plumbed 3-manifold M we construct from its plumbed graph a graded $\mathbb{Z}[U]$-module. This, for rational homology spheres, conjecturally equals the Heegaard-Floer homology of Ozsváth and Szabó, but it has even more structure. In particular, it shares several properties of the Heegaard-Floer homology, for example its normalized Euler-characteristic is the Seiberg-Witten invariant. If M is a complex surface singularity link then the new invariant can be compared with those coming from the analytic geometry. The talk will emphasize some of the intriguing connections of the analytic and topological invariants in the light of this new object. For example, we get new characterizations of rational and elliptic singularities, a new guiding principle for classification, or description of geometric genus (and other sheaf cohomologies) in terms of Seiberg-Witten invariant (subject of the Seiberg-Witten Invariant Conjecture of Nicolaescu and the author). We list some further open problems and conjectures as well.

A topological index theorem for manifolds with corners

Victor Nistor

We define a topological and an analytical index for manifolds with corners M. They both live in the K-theory groups $K_0(C_b^*(M))$ of the groupoid algebra associated to our manifold with corners M by integrating the Lie algebra of vector fields tangent to all faces of M. We prove that the topological and analytic index coincide. For M smooth (no corners), this is the Atiyah-Singer index theorem. If all the open faces of M are euclidean spaces, then the index maps are isomophisms, which gives a way of computing the K-theory of the groups $K_*(C_b^*(M))$. The proof uses a double-deformation groupoid obtained by integrating a suitable Lie algebroid. This is a joint work with Bertrand Monthubert.

The signature package on Witt spaces

Paolo Piazza

Let X be an orientable closed compact riemannian manifold with fundamental group G. Let X' be a Galois G-covering and r: $X \to BG$ a classifying map for X'. The signature package for $(X, r : X \to BG)$ can be informally stated as follows:

- there is a signature index class in the K-theory of the reduced C^*-algebra of G
- the signature index class is a bordism invariant
- the signature index class is equal to the C^*-algebraic Mishchenko signature, also a bordism invariant which is, in addition, a homotopy invariant
- there is a K-homology signature class in $K^*(X)$ whose Chern character is, rationally, the Poincare' dual of the L-Class
- if the assembly map in K-theory is rationally injective one deduces from the above results the homotopy invariance of Novikov higher signatures

The goal of my talk is to discuss the signature package on a class of stratified psedomanifolds known as Witt spaces. The topological objects involve intersection homology and Siegel's Witt bordism groups. The analytic objects involve some delicate elliptic theory on the regular part of the stratified pseudomanifold. Our analytic results reestablish (with completely different techniques) and extend results of Jeff Cheeger. This is joint work, some still in progress, with Pierre Albin, Eric Leichtnam and Rafe Mazzeo.

Group measure space decomposition of factors and W^*-superrigidity

Sorin Popa

A free ergodic measure preserving action of a countable group on a probability space, $\Gamma \curvearrowright X$, gives rise to a II_1 factor, $L^\infty(X) \rtimes \Gamma$, through the *group measure space* construction of Murray and von Neumann. In general, much of the initial data $\Gamma \curvearrowright X$ is "forgotten" by the isomorphism class of $L^\infty(X) \rtimes \Gamma$, for instance all free ergodic probability measure preserving actions of amenable groups give rise to isomorphic II_1 factors (Connes 1975). But a rich and deep rigidity theory underlies the non-amenable case. For instance, I have shown in 2005 that any isomorphism of II_1 factors associated with Bernoulli actions $\Gamma \curvearrowright X$, $\Lambda \curvearrowright Y$, of Kazhdan groups Γ, Λ comes from a conjugacy of the actions (W^*-*strong rigidity*). I will present a recent joint work with Stefaan Vaes, in which we succeeded to prove a W^*-*superrigidity* result for Bernoulli (+ other) actions $\Gamma \curvearrowright X$ of amalgamated free product groups $\Gamma = \Gamma_1 *_\Sigma \Gamma_2$, where Γ_1 is Kazhdan and Σ infinite amenable satisfying a combination of singularity/normality conditions in Γ_1, Γ_2. It shows that any isomorphism between $L^\infty(X) \rtimes \Gamma$ and a factor $L^\infty(Y) \rtimes \Lambda$ arising from an *arbitrary* free ergodic action $\Lambda \curvearrowright Y$, comes from a conjugacy of $\Gamma \curvearrowright X$, $\Lambda \curvearrowright Y$.

Noncommutative residues and projections associated to boundary value problems

Elmar Schrohe

On a compact manifold X with boundary we consider the realization $B = P_T$ of an elliptic boundary problem, consisting of a differential operator P and a differential boundary condition T. We assume that B is parameter-elliptic in small sectors around two rays in the complex plane, say $\arg\lambda = \phi$ and $\arg\lambda = \theta$. Associated to the cuts along the rays one can then define two zeta functions ζ_ϕ and ζ_θ for B. Both extend to meromorphic functions on the plane; the origin is a regular point. We relate the difference of the values at the origin to a noncommutative residue for the associated spectral projection $\Pi_{\phi,\theta}(B)$ defined by

$$\Pi_{\phi,\theta} u = \frac{i}{2\pi} \int_{\Gamma_{\theta,\phi}} \lambda^{-1} B(B-\lambda)^{-1} u \, d\lambda, \quad u \in \mathrm{dom}(B)$$

where $\Gamma_{\theta,\phi}$ is the contour which runs on the first ray from infinity to $r_0 e^{i\phi}$ for some $r_0 > 0$, then clockwise about the origin on the circle of radius r_0 to $r_0 e^{i\theta}$ and back to infinity along the second ray.

Fine structure of special symplectic spaces

Robert J. Stanton

Riemannian symmetric spaces can be classified according to $\mathbb{Z}_3$ and $\mathbb{Z}_5$ gradings of Lie algebras. The $\mathbb{Z}_5$ gradings give rise to special symplectic spaces. Using symplectic methods we give a rather complete description of the orbit structure of special symplectic spaces. Applications to realizations of special geometric structures, to Lie theory, and to new composition structures on orbits will be presented. This is joint with M. Slupinski.

Operations on Hochschild complexes

Boris Tsygan

The subject of operations on Hochschild and cyclic complexes of algebras is surprisingly rich and difficult. It is far from being settled after at least twenty years of study by many authors. I will try to outline its current state, including results of Kontsevich and Soibelman, of myself and Bressler, Nest, Dolgushev and Tamarkin, of Costello, and of Lurie.

The index of projective families of elliptic operators

Mathai Varghese

I will talk about ongoing research with Melrose and Singer, where we recently established an index theorem for projective families of elliptic operators. In this context, the index takes values in a smooth version of the twisted K-theory of the parametrizing space.

Aspects of free analysis

Dan Voiculescu

Motivated by free probability questions, the lecture will deal with duality for the bialgebra of the free difference quotient derivation and the highly noncommutative generalization of the Riemann sphere which appears in this context.

Algebras of p-symbols, noncommutative p-residue, and the Brauer group

Mariusz Wodzicki

Importance of the pseudodifferential symbol calculus extends far beyond the fundamental role it is known to play in Global and Microlocal Analysis. In this article, we demonstrate that algebras of symbols contribute to subtle phenomena in characteristic $p > 0$.

Geometric complexity and topological rigidity

Guoilang Yu

In this talk, I will introduce a notion of geometric complexity and discuss its applications to geometric group theory and rigidity of manifolds. In particular, I will show how to prove various geometric versions of the Borel conjecture under certain a finiteness condition on the geometric complexity. This is joint work with Erik Guentner and Romain Tessera.

List of Participants

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Henri Moscovici has made a number of major contributions to noncommutative geometry, global analysis, and representation theory. This volume, which includes articles by some of the leading experts in these fields, provides a panoramic view of the interactions of noncommutative geometry with a variety of areas of mathematics. It focuses on geometry, analysis and topology of manifolds and singular spaces, index theory, group representation theory, connections of noncommutative geometry with number theory and arithmetic geometry, Hopf algebras and their cyclic cohomology.

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