CONTEMPORARY MATHEMATICS

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Dynamical Systems and Group Actions

Lewis Bowen
Rostislav Grigorchuk
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Editors

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Dedicated with the greatest respect and admiration to Anatoli Stepin
on the occasion of his 70th birthday.

Contents

Preface

On July 20, 2010 Anatoli Mikhailovich Stepin, Professor of Mathematics at Moscow (Lomonosov) State University, celebrated his seventieth birthday. Anatoli Stepin studied mathematics at Moscow State University, then worked there for over 40 years. His supervisors were Feliks Berezin and Yakov Sinai and that shaped the range of research interests of Stepin: dynamical systems and ergodic theory as well as related problems of functional analysis and probability theory. A research direction of special interest has been the study of group actions on measured spaces, orbit equivalence, and σ-algebras.

It would take too long to describe all of the areas and problems to which Stepin contributed. We are going to mention very briefly just a few (for a more detailed account of his work, see [**1**]). Anatoli Stepin began his scientific career studying approximations of dynamical systems by periodic transformations. The pioneering results in this direction obtained jointly with Anatole Katok formed the basis of his Ph.D. thesis defended in 1968. Those results are now classical, they are widely and commonly used in ergodic theory and its applications. In 1967 Stepin received the Prize of the Moscow Mathematical Society, one of the major awards in Soviet mathematics.

Another one of the early results of Stepin was solution of a problem, due to Kolmogorov, on the group property of spectra of dynamical systems. His construction of dense measures with mutually singular convolutions found its applications in harmonic analysis and ergodic theory. The paper [**2**], written jointly with Anatole Katok, among other things laid the foundation of the theory of interval exchange transformations, which is now a rapidly developing direction in one-dimensional dynamics. Stepin proposed a cohomological approach to the study of orbit equivalence in [**3**]. Of great importance are results of Stepin and his students on the theory of billiard systems. He established and studied mechanisms of creating periodic trajectories in billiard systems without focusing and scattering. Stepin clarified the fundamental role of the Lyapunov exponents in the spectral theory of the weighted shift operators. He has results on the von Neumann algebras and C^*-algebras associated with dynamical systems. Yet another important contribution is the work of Stepin and his students on integrability of dynamical systems. Stepin was one of those who started the study of models of statistical physics, in particular, the Ising model on graphs and groups.

Stepin obtained a complete solution of the problem of calculating unitary invariants of induced flows on homogeneous spaces of semisimple and soluble groups, the problem having been posed as a consequence of the famous paper by Gelfand and Fomin.

The group actions and the use of symmetries were fundamental principles in Stepin's approach to many problems of contemporary mathematics. As early as 1970 he obtained results on the classification of decreasing sequences of homogeneous partitions and invented an entropy-type invariant for such sequences. These methods were further developed by Stepin and Vershik along with the idea of a local isomorphism that led to the introduction of the classes of hyperfinite groups and sofic groups, as well as to the use of various topologies in spaces of groups and subgroups to solve problems in group theory.

The paper [**4**] initiated the study of isomorphisms of Bernoulli shifts on non-commutative groups. The notion of an Ornstein group was introduced there, which is the subject of one of the papers in this volume. Stepin pioneered the use of invariant means on groups in the study of dynamical systems and statistical physics. In the early 70s, he was one of those who came up with an idea of a random ordering on a group and used it to prove variational type theorems for actions of non-commutative groups. The work of Stepin and his students led to a much better understanding of the class of amenable groups and amenable actions.

During his 40 years at the Department of Mathematics and Mechanics of Moscow State University, Anatoli Stepin supervised more than 30 Ph.D. students. Since 1978, he has been organizing (together with Dmitri Anosov and, from 1985 to 2002, Rostislav Grigorchuk) the research seminar on dynamical systems and ergodic theory at Moscow State University. The colleagues and students of Anatoli Stepin enjoyed working with him and praised him for his positive attitude, supportiveness and work ethic. The students can recall numerous field trips and sports tournaments involving seminars of Sinai, Kirillov and Stepin.

This collection of papers conceived and organized by the undersigned (of whom two are Anatoli Stepin's students) reflects to a large extent the research interests of Stepin. Many of the papers further develop or discuss ideas pioneered by Anatoli Stepin and it is a great pleasure for us to dedicate to him this volume of Contemporary Mathematics.

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Dynamical Systems and Group Actions • Bowen et al., Editors

This volume contains cutting-edge research from leading experts in ergodic theory, dynamical systems and group actions. A large part of the volume addresses various aspects of ergodic theory of general group actions including local entropy theory, universal minimal spaces, minimal models and rank one transformations. Other papers deal with interval exchange transformations, hyperbolic dynamics, transfer operators, amenable actions and group actions on graphs.

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