CONTEMPORARY MATHEMATICS

572

Computational Algebraic and Analytic Geometry

AMS Special Sessions on
Computational Algebraic and Analytic Geometry
for Low-Dimensional Varieties
January 8, 2007: New Orleans, LA
January 6, 2009: Washington, DC
January 6, 2011: New Orleans, LA

Mika Seppälä
Emil Volcheck
Editors

American Mathematical Society

Computational Algebraic and Analytic Geometry

CONTEMPORARY MATHEMATICS

572

Computational Algebraic and Analytic Geometry

AMS Special Sessions on
Computational Algebraic and Analytic Geometry
for Low-Dimensional Varieties
January 8, 2007: New Orleans, LA
January 6, 2009: Washington, DC
January 6, 2011: New Orleans, LA

Mika Seppälä
Emil Volcheck
Editors

American Mathematical Society
Providence, Rhode Island

2010 *Mathematics Subject Classification.* Primary 14HXX, 30FXX, and 68WXX.

Library of Congress Cataloging-in-Publication Data

Computational algebraic and analytic geometry : AMS special sessions on computational algebraic and analytic geometry for low-dimensional varieties, January 8, 2007, New Orleans, LA, January 6, 2009, Washington, DC, [and] January 6, 2011, New Orleans, LA / Mika Seppälä, Emil Volcheck, editors.

p. cm. — (Contemporary mathematics ; v. 572)

Includes bibliographical references.

ISBN 978-0-8218-6869-0 (alk. paper)

1. Curves, Algebraic–Data processing–Congresses. 2. Riemann surfaces–Congresses. I. Seppälä, Mika. II. Volcheck, Emil, 1966–

QA565.C658 2012

512'.5–dc23 2012009188

---

10 9 8 7 6 5 4 3 2 1   17 16 15 14 13 12

Contents

Preface

Distinct communities of mathematicians have grown around analytical and algebraic approaches to geometry. Even though both approaches are deeply connected through results such as Chow's Theorem and GAGA, mathematicians infrequently collaborate across these communities. Computational methods make these connections explicit and increase our understanding of geometry in ways not possible when each approach is pursued as its own form of pure mathematics. In this way, computational methods help bring together these different communities of mathematicians.

Uniformization of Riemann surfaces is a prime example of a topic where computational methods are bringing important new insights. In the late 19th and early 20th centuries, mathematicians such as Burnside, Koebe, Myrberg, Rankin, and Whittaker labored to make uniformization explicit, developing numerical techniques even when no computers were available. Their work is considered to be a crowning achievement of geometry in that era. The Abel-Jacobi and Torelli Theorems represent another prominent example of a theory that relates analytical and algebraic representations of geometric objects, in this case, relating complex lattices to curves and their Jacobians.

During the last twenty years, practical numerical and symbolic computations have become commonplace and possible for anybody. This has given new life to some of the old ideas, and has led to new approaches to some of the classical problems. Here are some examples of such work.

Uniformization has advanced in both theory and practice through the development of effective computational methods for special cases of the problem. Ideally one would like to find explicit symbolic methods to pass, for example, from an algebraic plane curve, given by a polynomial, to a uniformization of the curve in question. In the case of genus one, the symbolic approach is part of the classical analysis of elliptic curves. For curves of higher genus, symbolic methods have succeeded in special cases only. Numeric methods have yielded more general results, but a solution to the general case still looms far in the future. These numeric methods lead one to study, for example, algebraic curves given by approximations of the actual polynomials defining the curve, which is a major topic in numerical algebraic geometry.

An example of a completely new theory whose development was supported by computational methods is a discrete version of the Riemann mapping theorem offered by circle packings. Research on explicit methods connecting Riemann surfaces and their corresponding Fuchsian groups has also benefited from computational methods. This series is also inspired by work of Curtis McMullen, who, in his AMS Colloquium Lectures in 2000, connected dynamics on a Riemann surface to rational points on the corresponding algebraic curve.

This volume is a collection of research papers on computational methods in algebraic and analytic geometry. It has its roots in the series of AMS Special Sessions on Computational Algebraic and Analytic Geometry that have taken place at the Joint AMS-MAA National Meetings every odd year since 1999, and in the large European research projects that coordinated the work in this area during 1991–1996.

Usually AMS Special Sessions and other similar meetings are characterized by the methods used in the papers presented. This volume and the preceding AMS Special Sessions form an exception to this rule: papers published here and those presented earlier in the Special Sessions entitled Computational Algebraic and Analytic Geometry on Low-Dimensional Varieties have, as their unifying factor, the same object of study.

Compact Riemann surfaces are algebraic curves. They are also characterized by their Jacobian variety and their Fuchsian group. Hence the same object can be studied by a variety of methods: analytic, algebraic, and geometric. It is this extraordinary variety of methods that makes this area challenging, interesting, and very fertile.

The editors are grateful for the contributions of the authors, and the referees who helped to create this volume. The editors thank all the sponsoring institutions that helped to advance research in this field. Most importantly, the editors thank the American Mathematical Society and its expert publishing officers.

Mika Seppälä and Emil Volcheck

CONM 572

Computational Algebraic and Analytic Geometry • Seppälä et al., Editors

This volume contains the proceedings of three AMS Special Sessions on Computational Algebraic and Analytic Geometry for Low-Dimensional Varieties held January 8, 2007, in New Orleans, LA; January 6, 2009, in Washington, DC; and January 6, 2011, in New Orleans, LA.

Algebraic, analytic, and geometric methods are used to study algebraic curves and Riemann surfaces from a variety of points of view. The object of the study is the same. The methods are different. The fact that a multitude of methods, stemming from very different mathematical cultures, can be used to study the same objects makes this area both fascinating and challenging.

ISBN 978-0-8218-6869-0

9 780821 868690

CONM/572

AMS