CONTEMPORARY MATHEMATICS

576

Groups and Model Theory

In Honor of Rüdiger Göbel's 70th Birthday May 30–June 3, 2011 Conference center "Die Wolfsburg", Mülheim an der Ruhr, Germany

> Lutz Strüngmann Manfred Droste László Fuchs Katrin Tent Editors



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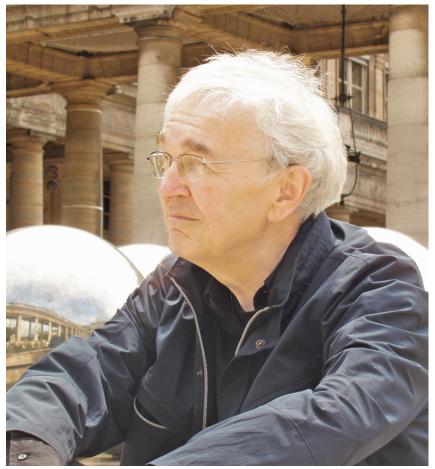
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This volume is dedicated to Professor Rüdiger Göbel on the occasion of his 70^{th} birthday

Rüdiger in the yard of the Grand Palais, Paris 2008

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About the conference

The international conference on Groups and Model Theory was held in honor of Rüdiger Göbel on the occasion of his 70th birthday, at the Catholic academy 'Die Wolfsburg,' Mülheim an der Ruhr (Germany), in June 2011. More than eighty scientists from Australia, the Czech Republic, China, England, Germany, Ireland, Israel, Italy, Russia, Scotland, Spain, Turkey, and the United States of America participated and enjoyed mathematics in the beautiful location.

Group theory and model theory were chosen as the topics of the conference, reflecting Rüdiger's mathematical interests. Little more than three decades have passed since algebra and, in particular group theory, received new impetus from the applications of advanced methods of logic to algebraic questions. In particular, model theory has strongly influenced group theory, both commutative and noncommutative. This led to striking new developments in group theory as a field of application and had an interesting feedback to model theory. Since then this interplay has fruitfully been revisited by algebraists and model theorists alike. It is being studied extensively by a broad community and will surely have a strong and promising future. Rüdiger is one of the pioneers in this interplay between algebra and model theory.

The conference focused on the interdisciplinary relation between these two fields of research. The invited speakers - Bazzoni, Fuchs, Glass, Göbel, Macpherson, Salce, Shelah, Thomas, Trlifaj and Truss - are leading experts in their fields of research. The organizers were especially delighted to have Saharon Shelah as a speaker and his participation was a great asset to the conference. Moreover, several young scientists and students attended the conference and took the opportunity to present their results and to meet leading scientists for discussions on the most recent developments in their fields of interest.

The articles in this volume deal with abelian groups, modules over commutative rings, permutation groups, automorphism groups of homogeneous structures like graphs, relational structures, geometries, topological spaces or groups, consequences of model theoretic properties like stability or categoricity, subgroups of small index, the lattice of normal subgroups and simplicity, the automorphism tower problem, algebraic entropy as well as random constructions. All the articles were refereed according to strict standards.

The conference was supported by generous grants from the German Research Foundation, the Universities of Duisburg-Essen and Münster, the University of Applied Sciences Mannheim; technical support was provided by Enigma Software. We gratefully acknowledge all their support.

ABOUT THE CONFERENCE

The conference was organised by Manfred Droste, Lutz Strüngmann, Katrin Tent, and Martin Ziegler. In particular, we would like to thank Katrin Leistner for the tremendous administrative work she has done and our students Ruth Heselhaus and Malte Heitbrede for their help in organizing a successful conference. Special thanks are due to Heidi Göbel for organizing daily activities for the attending spouses of the participants.

Manfred Droste, László Fuchs, Lutz Strüngmann, and Katrin Tent

A few words about Rüdiger

Rüdiger Göbel was born on 27th December, 1940 in Fürstenwalde, and spent his early youth in the former German Democratic Republic. His family moved to West Germany, where he studied physics and mathematics at Johann W. Goethe University in Frankfurt am Main. Having found his passion for mathematics and theoretical physics, Rüdiger was heavily influenced and inspired by Reinhold Baer who guided him to his PhD which he received in 1967. They had a close relationship, but with the special flavor of the 'old school.' As Rüdiger told us, even long after he became full professor, they still greeted each other as "Guten Morgen, Herr Professor Baer" — "Guten Morgen, Herr Göbel."

After assistant professorships in Würzburg (Germany) and Austin (Texas) working in relativity theory and general physics, he obtained his Habilitation with a thesis on General Relativity Theory and Group Theory in 1974. In the same year he got a full professorship at the University of Essen (which became later the University of Duisburg-Essen) where he stayed until his recent retirement. Being a very active person, Rüdiger travelled a lot, lecturing at a large number of universities, and giving talks at various conferences. He also held positions as visiting professor in Dortmund, London, Las Cruces, Waco, Jerusalem, and Middletown. He was the main organizer of a number of conferences, co-founder of Forum Mathematicum, and an editor of various proceedings.

It is almost impossible to describe all of Rüdiger's work and contributions to mathematics as well as to physics and computer science; this page has certainly not enough space to do so. We just want to point out that his prolific research activity can be seen from his over 200 papers in prominent international journals (5 of them are still in progress). His research text with Jan Trlifaj has become a classic in the field. In his publications, he solved several open problems in group theory, initiated new theories, and his ideas stimulated a large amount of new research.

Shortly after Rüdiger moved to Essen, he got interested in Abelian Group Theory and, in particular, in applications of set-theoretic and model-theoretic methods to Abelian groups. Soon he became a leading expert in this area and started his fruitful collaboration with Saharon Shelah. Several projects sponsored by the German-Israeli Foundation enabled Rüdiger and Saharon to attack some of the hardest problems in Abelian Group Theory and general Module Theory over commutative rings. Their joint forces led to extraordinary results. His collaboration with more than fifty coauthors produced important results in several other fields as well. He is a hard and careful worker, very knowledgeable and quick, besides being friendly and polite; it is a real pleasure to work with him. To his coauthors and to many researchers world-wide Rüdiger is not just a colleague, but also a close, highly-respected friend. He developed deep friendships with numerous colleagues and the hospitality of the Göbel home to friends, colleagues, visitors, even to students is legendary.

No picture of Rüdiger would be complete without pointing out his exemplary relationship to his students. His lectures had a particular style with partially ordered proofs rather than linearly ordered ones, but always with a lot of enthusiasm and passion in his explanations (we may claim so, since two of the editors were his students). From a student's point of view, he was sometimes very challenging (e.g. when proving the equivalence of Zorn's lemma and the axiom of choice in a first year linear algebra course), but this was his special way of showing the students the beauty of mathematics, in particular, of algebra, and of motivating and even of inspiring them. Rüdiger has always been available for students in a friendly and encouraging manner. He is popular as a research professor as he respects the students, knows how to help them, and is very generous in sharing his ideas. Over the years he attracted many students, and guided them through their diploma theses. He had more than twenty Ph.D. students, six of whom are now full professors.

When he was asked: "What part of your research are you most proud of?", Rüdiger reached back to his roots and answered: "A sentence in a publication by Stephen Hawking thanking me for a seminar that I gave to students at Cambridge University." Seemingly this inspired Hawking (who attended the talk) to write one of his essential papers on Zeeman's conjecture. He was proud that his attempt to make the students understand something on relativity theory inspired a person like Stephen Hawking.

Last, but not least, we should mention that such a successful career would not have been possible without his wife Heidi, whom he married in 1969. Her loving and caring support during the four decades of their marriage and the warmth she showed to the countless visitors to the Göbel family home, played a major role in supporting Rüdiger's extraordinary accomplishments. Rüdiger has always been very close to his daughter, Ines, who has followed her father's footsteps in also studying mathematics. This closeness can be seen from the fact that Rüdiger persuaded his co-authors to adopt the notion of Ines(sential) homomorphisms in Abelian group theory. In recent years Ines has introduced Rüdiger to a new nonacademic interest: sailing. No doubt he will bring the same passion to this that he has shown for mathematics!

It is a great pleasure to see that Rüdiger keeps working with a youthful spirit and a great deal of enthusiasm. On the occasion of his 70th birthday all his friends, coauthors and students join us in wishing him good health to continue his activities for many more years to come.

Lieber Rüdiger, wir wünschen Dir alles Gute für die Zukunft!

Katrin, László, Lutz, and Manfred



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This book contains the proceedings of the conference "Groups and Model Theory", held May 30–June 3, 2011, in Mülheim an der Ruhr, Germany, in honor of Rüdiger Göbel's 70th birthday.

In the last thirty years, group theory has received new input through the application of methods from logic to problems in algebra. In particular, model theory has strongly influenced both commutative and non-commutative group theory. This led to striking new developments in group theory and has had an interesting impact back on model theory. This interplay has been revisited by algebraists and model theorists and is showing strong and promising roads for future research.

This book presents important current research at the border of model theory and group theory by renowned researchers. Articles in this volume cover abelian groups, modules over commutative rings, permutation groups, automorphism groups of homogeneous structures such as graphs, relational structures, geometries, topological spaces or groups, consequences of model theoretic properties like stability or categoricity, subgroups of small index, the automorphism tower problem, as well as random constructions.



