CONTEMPORARY MATHEMATICS

581

# Recent Advances in Harmonic Analysis and Partial Differential Equations

AMS Special Sessions
March 12–13, 2011
Statesboro, Georgia

The JAMI Conference
March 21–25, 2011
Baltimore, Maryland

Andrea R. Nahmod
Christopher D. Sogge
Xiaoyi Zhang
Shijun Zheng
Editors

American Mathematical Society

# Recent Advances in Harmonic Analysis and Partial Differential Equations

# CONTEMPORARY MATHEMATICS

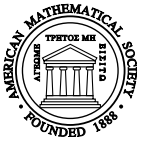
581

# Recent Advances in Harmonic Analysis and Partial Differential Equations

AMS Special Sessions
March 12–13, 2011
Statesboro, Georgia

The JAMI Conference
March 21–25, 2011
Baltimore, Maryland

Andrea R. Nahmod
Christopher D. Sogge
Xiaoyi Zhang
Shijun Zheng
Editors




**American Mathematical Society**
Providence, Rhode Island

**EDITORIAL COMMITTEE**

Dennis DeTurck, managing editor

George Andrews Abel Klein Martin J. Strauss

2000 *Mathematics Subject Classification.* Primary 42B37, 35J10, 35K52, 35Q55, 35L70, 58J45; Secondary 74J30, 76D05.

---

**Library of Congress Cataloging-in-Publication Data**

Recent advances in harmonic analysis and partial differential equations : AMS special sessions, March 12–13, 2011, Statesboro, Georgia : the JAMI Conference, March 21–25, 2011, Baltimore, Maryland / Andrea R. Nahmod, Christopher D. Sogge, Xiaoyi Zhang, Shijun Zheng, editors.

p. cm — (Contemporary mathematics ; v. 581)

Includes bibliographical references.

ISBN 978-0-8218-6921-5 (alk. paper)

1. Harmonic analysis–Congresses. 2. Differential equations, Partial–Congresses. I. Nahmod, Andrea R., 1964-

QA403.R425 2012

2012023439

---

**Copying and reprinting.** Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to `reprint-permission@ams.org`.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2012 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights
except those granted to the United States Government.
Copyright of individual articles may revert to the public domain 28 years
after publication. Contact the AMS for copyright status of individual articles.
Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 17 16 15 14 13 12

## Contents

Preface vii

Combined list of speakers ix

The defocusing cubic nonlinear wave equation in the energy-supercritical regime
Aynur Bulut 1

Stein's extension operator on weighted Sobolev spaces on Lipschitz domains and applications to interpolation
Kevin Brewster, Irina Mitrea, and Marius Mitrea 13

Multilinear Morawetz identities for the Gross-Pitaevskii hierarchy
T. Chen, N. Pavlović, and N. Tzirakis 39

Elementary proofs for Kato smoothing estimates of Schrödinger-like dispersive equations
Xuwen Chen 63

The conormal derivative problem for higher order elliptic systems with irregular coefficients
Hongjie Dong and Doyoon Kim 69

Localization of eigenfunctions of a one-dimensional elliptic operator
Marcel Filoche, Svitlana Mayboroda, and Brandon Patterson 99

Scattering and blow up for the two-dimensional focusing quintic nonlinear Schrödinger equation
Cristi Guevara and Fernando Carreon 117

Transverse stability of periodic traveling waves in Kadomtsev-Petviashvili equations: A numerical study
Christian Klein and Christof Sparber 155

Time decay for the solutions of a fourth-order nonlinear Schrödinger equation
Jeng-Eng Lin 169

Self-adjoint realizations of Schrödinger operators on vector bundles over Riemannian manifolds
Ognjen Milatovic 175

Spectral properties of the reflection operator in two dimensions
Irina Mitrea, Katharine Ott, and Eric Stachura 199

Recent local and global solutions to the Lagrangian averaged Navier-Stokes equation
Nathan Pennington 217

Recent works on the Strauss conjecture
Chengbo Wang and Xin Yu 235

Wave operators for nonlinear wave equations with null structures
Dong Li and Xiaoyi Zhang 257

Fractional regularity for nonlinear Schrödinger equations with magnetic fields
Shijun Zheng 271

# Preface

In the past decades there has been increasing interest in the study of partial differential equations by using harmonic analysis methods and techniques. The publication of this special volume of *Contemporary Mathematics* is motivated by two concurrent AMS special sessions on this subject (*Harmonic Analysis and PDEs* and *Nonlinear Analysis of PDEs*) at Georgia Southern University, March 12-13, 2011, and the JAMI (Japan-U.S. Mathematics Institute) conference at Johns Hopkins University, March 21-25, 2011. These two conferences' theme centers on current research in Analysis and PDE, with an emphasis on the interaction between them.

This proceeding features a collection of invited contributions of survey and research articles authored by some of the active and recognized experts in the area. The topics mainly focus on using Fourier, spectral and geometrical methods to treat wellposedness, regularity, scattering and stability problems in PDE, which include dispersive type evolution equations, higher-order systems and Sobolev spaces theory that arise in aspects of mathematical physics. These involve state-of–the-art techniques and approaches that have been used and developed in the last few years. The theory and the tools are interrelated, which reflect some of the deep connections between various subjects in both classical and modern analysis.

The paper of H. Dong and D. Kim considers the conormal boundary problem for higher order elliptic systems with irregular complex-valued coefficients. K. Brewster, I. Mitrea and M. Mitrea study extension and interpolation properties of certain weighted Sobolev spaces on Lipschitz domain and general manifolds. Wellposedness and blowup results for nonlinear dispersive equations are addressed in the papers of A. Bulut, T. Chen, N. Pavlovic and N. Tzirakis, C. Guevara and F. Carreon, and S. Zheng. Crucial smoothing and local energy decay inequalities are proved in X. Chen and J.-E. Lin's papers. Scattering problem for 3D wave equations with critical nonlinearities are considered in the paper of D. Li and X. Zhang. Analytical and numeral results with applications for elliptic and Kadomtsev-Petviashvili equations are studied in M. Filoche, S. Mayboroda and B. Patterson, and C. Klein and C. Sparber's papers.

C. Wang and X. Yu give a review on recent progress on Strauss conjecture for wave equations on exterior domains. N. Pennington gives an overview on the local and global solutions of the Lagrangian averaged Navier-Stokes equation. Spectral properties of certain singular integral operator are studied in the paper of I. Mitrea, K. Ott and E. Stachura. O. Milatovic studies the essential self-adjointness of Schrödinger operators on certain Riemannian manifolds. All manuscripts in this volume are peer-reviewed.

This book aims at providing researchers a valuable reference for their current and future investigations on similar problems. It might also serve as an update

inspiring literature for graduate students or young mathematicians in studying the subjects and pursuing the path that might lead to finding a breakthrough in a relevant field.

We would like to thank the American Mathematical Society for helping organize and sponsor the AMS Sectional Southeastern Conference. We thank Georgia Southern University for supporting and sponsoring the special sessions. We thank Johns Hopkins University for sponsoring the JAMI Conference. We thank all the participants of the conferences, including young and senior mathematicians, recent doctorates as well as leading experts for their enthusiasm and support.

The Editors: Andrea R. Nahmod, Christopher D. Sogge, Xiaoyi Zhang and Shijun Zheng, Principal Editor

## Combined list of speakers

The invited talks focused on perturbation theory (differential, geometric or probabilistic), wellposedness, blowup, scattering and stability problems for linear and nonlinear partial differential equations that mainly arise in Quantum Field Theory, General Relativity as well as Fluid Dynamics. The theory and the methods applied address perspectives of the advances in relevant areas.

Here are some links to the AMS special sessions and the JAMI conference.
http://www.ams.org/meetings/sectional/2173_program_ss1.html#title
http://www.ams.org/meetings/sectional/2173_program_ss15.html#title
http://www.mathematics.jhu.edu/new/jami2011/analysispde.htm

The following are lists of the names of the organizers and their speakers.

AMS special session organizers: Paul Hagelstein, Baylor University
Ronghua Pan, Georgia Institute of Technology
Alexander Stokolos, GSU
Xiaoyi Zhang, IAS and University of Iowa
Shijun Zheng, GSU

Speakers at **Harmonic Analysis and PDEs:**

Matthew Blair, University of New Mexico
Russell Brown, University of Kentucky
Aynur Bulut, University of Texas, Austin
Hans Christianson, UNC
Michael Goldberg, University of Cincinnati
William Green, EIU
Alex Iosevich, University of Rochester
Yulia Karpechina, UAB
Xiaosheng Li, FIU
Svitlana Mayboroda, Purdue
Jason Metcalfe, UNC
Andrea Nahmod, University of Massachusetts
Konstantin Oskolkov, University of South Carolina
Katharine Ott, University of Kentucky
Benoit Pausader, Brown University
Cristian Rios, University of Calgary
Gideon Simpson, University of Toronto
Xin Yu, JHU

Speakers at **Nonlinear Analysis of PDEs:**

Hongjie Dong, Brown University
Gung-Min Gie, University of California, Riverside
Xianpeng Hu, NYU
J.-E. Lin, George Mason University
Zhiwu Lin, GA Tech
Nathan Pennington, Kansas State University
Alexander B. Reznikov, Michigan State University
Fernando Schwartz, University of Tennessee
Christof Sparber, University of Illinois, Chicago
Erwin Suazo, University of Puerto Rico
Andrzej Swiech, GA Tech
Xiangjin Xu, Binghamton University, SUNY
Yanni Zeng, UAB
Chunshan Zhao, GSU

Organizers and Speakers at the **JAMI Conference: Analysis of PDEs**

Organizing Committee: Hans Lindblad (UC San Diego and JHU)
Christopher Sogge (JHU), Chengbo Wang (JHU)

Speakers: Lars Andersson, Max Plank
Marius Beceanu, Rutgers University
Pieter Blue, University of Edinburgh
Nicolas Burq, Universite Paris-Sud
Jean-Marc Delort, Universite Paris Nord
Benjamin Dodson, University of California, Berkeley
Daoyuan Fang, Zhejiang University
Manoussos Grillakis, University of Maryland
Hideo Kozono, Tohoku University
Jason Metcalfe, University of North Carolina
Makoto Nakamura, Tohoku University
Igor Rodnianski, Princeton University
Shuanglin Shao, University of Minnesota
Avy Soffer, Rutgers University
Daniel Tataru, University of California, Berkeley
Tai-Peng Tsai, University of British Columbia
Sijue Wu, University of Michigan
Xiaoyi Zhang, University of Iowa

# Selected Published Titles in This Series

581 **Andrea R. Nahmod, Christopher D. Sogge, Xiaoyi Zhang, and Shijun Zheng, Editors,** Recent Advances in Harmonic Analysis and Partial Differential Equations, 2012

571 **José Ignacio Burgos Gil, Rob de Jeu, James D. Lewis, Juan Carlos Naranjo, Wayne Raskind, and Xavier Xarles, Editors,** Regulators, 2012

569 **Victor Goryunov, Kevin Houston, and Roberta Wik-Atique, Editors,** Real and Complex Singularities, 2012

568 **Simeon Reich and Alexander J. Zaslavski, Editors,** Optimization Theory and Related Topics, 2012

567 **Lewis Bowen, Rostislav Grigorchuk, and Yaroslav Vorobets, Editors,** Dynamical Systems and Group Actions, 2012

566 **Antonio Campillo, Gabriel Cardona, Alejandro Melle-Hernández, Wim Veys, and Wilson A. Zúñiga-Galindo, Editors,** Zeta Functions in Algebra and Geometry, 2012

565 **Susumu Ariki, Hiraku Nakajima, Yoshihisa Saito, Ken-ichi Shinoda, Toshiaki Shoji, and Toshiyuki Tanisaki, Editors,** Algebraic Groups and Quantum Groups, 2012

564 **Valery Alexeev, Angela Gibney, Elham Izadi, János Kollár, and Eduard Looijenga, Editors,** Compact Moduli Spaces and Vector Bundles, 2012

563 **Primitivo B. Acosta-Humánez, Federico Finkel, Niky Kamran, and Peter J. Olver, Editors,** Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics, 2012

562 **P. Ara, K. A. Brown, T. H. Lenagan, E. S. Letzter, J. T. Stafford, and J. J. Zhang, Editors,** New Trends in Noncommutative Algebra, 2012

561 **Óscar Blasco, José A. Bonet, José M. Calabuig, and David Jornet, Editors,** Topics in Complex Analysis and Operator Theory, 2012

560 **Weiping Li, Loretta Bartolini, Jesse Johnson, Feng Luo, Robert Myers, and J. Hyam Rubinstein, Editors,** Topology and Geometry in Dimension Three, 2011

559 **Guillaume Bal, David Finch, Peter Kuchment, John Schotland, Plamen Stefanov, and Gunther Uhlmann, Editors,** Tomography and Inverse Transport Theory, 2011

558 **Martin Grohe and Johann A. Makowsky, Editors,** Model Theoretic Methods in Finite Combinatorics, 2011

557 **Jeffrey Adams, Bong Lian, and Siddhartha Sahi, Editors,** Representation Theory and Mathematical Physics, 2011

556 **Leonid Gurvits, Philippe Pébay, J. Maurice Rojas, and David Thompson, Editors,** Randomization, Relaxation, and Complexity in Polynomial Equation Solving, 2011

555 **Alberto Corso and Claudia Polini, Editors,** Commutative Algebra and Its Connections to Geometry, 2011

554 **Mark Agranovsky, Matania Ben-Artzi, Greg Galloway, Lavi Karp, Simeon Reich, David Shoikhet, Gilbert Weinstein, and Lawrence Zalcman, Editors,** Complex Analysis and Dynamical Systems IV, 2011

553 **Mark Agranovsky, Matania Ben-Artzi, Greg Galloway, Lavi Karp, Simeon Reich, David Shoikhet, Gilbert Weinstein, and Lawrence Zalcman, Editors,** Complex Analysis and Dynamical Systems IV, 2011

552 **Robert Sims and Daniel Ueltschi, Editors,** Entropy and the Quantum II, 2011

551 **Jesus Araujo-Gomez, Bertin Diarra, and Alain Escassut, Editors,** Advances in Non-Archimedean Analysis, 2011

550 **Y. Barkatou, S. Berhanu, A. Meziani, R. Meziani, and N. Mir, Editors,** Geometric Analysis of Several Complex Variables and Related Topics, 2011

549 **David Blázquez-Sanz, Juan J. Morales-Ruiz, and Jesús Rodríguez Lombardero, Editors,** Symmetries and Related Topics in Differential and Difference Equations, 2011

548 **Habib Ammari, Josselin Garnier, Hyeonbae Kang, and Knut Sølna, Editors,** Mathematical and Statistical Methods for Imaging, 2011

547 **Krzysztof Jarosz, Editor,** Function Spaces in Modern Analysis, 2011

546 **Alain Connes, Alexander Gorokhovsky, Matthias Lesch, Markus Pflaum, and Bahram Rangipour, Editors,** Noncommutative Geometry and Global Analysis, 2011

545 **Christian Houdré, Michel Ledoux, Emanuel Milman, and Mario Milman, Editors,** Concentration, Functional Inequalities and Isoperimetry, 2011

544 **Carina Boyallian, Esther Galina, and Linda Saal, Editors,** New Developments in Lie Theory and Its Applications, 2011

543 **Robert S. Doran, Paul J. Sally Jr., and Loren Spice, Editors,** Harmonic Analysis on Reductive, $p$-adic Groups, 2011

542 **E. Loubeau and S. Montaldo, Editors,** Harmonic Maps and Differential Geometry, 2011

541 **Abhijit Champanerkar, Oliver Dasbach, Efstratia Kalfagianni, Ilya Kofman, Walter Neumann, and Neal Stoltzfus, Editors,** Interactions Between Hyperbolic Geometry, Quantum Topology and Number Theory, 2011

540 **Denis Bonheure, Mabel Cuesta, Enrique J. Lami Dozo, Peter Takáč, Jean Van Schaftingen, and Michel Willem, Editors,** Nonlinear Elliptic Partial Differential Equations, 2011

539 **Kurusch Ebrahimi-Fard, Matilde Marcolli, and Walter D. van Suijlekom, Editors,** Combinatorics and Physics, 2011

538 **José Ignacio Cogolludo-Agustín and Eriko Hironaka, Editors,** Topology of Algebraic Varieties and Singularities, 2011

537 **César Polcino Milies, Editor,** Groups, Algebras and Applications, 2011

536 **Kazem Mahdavi, Deborah Koslover, and Leonard L. Brown III, Editors,** Cross Disciplinary Advances in Quantum Computing, 2011

535 **Maxim Braverman, Leonid Friedlander, Thomas Kappeler, Peter Kuchment, Peter Topalov, and Jonathan Weitsman, Editors,** Spectral Theory and Geometric Analysis, 2011

534 **Pere Ara, Fernando Lledó, and Francesc Perera, Editors,** Aspects of Operator Algebras and Applications, 2011

533 **L. Babinkostova, A. E. Caicedo, S. Geschke, and M. Scheepers, Editors,** Set Theory and Its Applications, 2011

532 **Sergiy Kolyada, Yuri Manin, Martin Möller, Pieter Moree, and Thomas Ward, Editors,** Dynamical Numbers: Interplay between Dynamical Systems and Number Theory, 2010

531 **Richard A. Brualdi, Samad Hedayat, Hadi Kharaghani, Gholamreza B. Khosrovshahi, and Shahriar Shahriari, Editors,** Combinatorics and Graphs, 2010

530 **Vitaly Bergelson, Andreas Blass, Mauro Di Nasso, and Renling Jin, Editors,** Ultrafilters across Mathematics, 2010

529 **Robert Sims and Daniel Ueltschi, Editors,** Entropy and the Quantum, 2010

528 **Alberto Farina and Enrico Valdinoci, Editors,** Symmetry for Elliptic PDEs, 2010

527 **Ricardo Castaño-Bernard, Yan Soibelman, and Ilia Zharkov, Editors,** Mirror Symmetry and Tropical Geometry, 2010

526 **Helge Holden and Kenneth H. Karlsen, Editors,** Nonlinear Partial Differential Equations and Hyperbolic Wave Phenomena, 2010

525 **Manuel D. Contreras and Santiago Díaz-Madrigal, Editors,** Five Lectures in Complex Analysis, 2010

524 **Mark L. Lewis, Gabriel Navarro, Donald S. Passman, and Thomas R. Wolf, Editors,** Character Theory of Finite Groups, 2010

For a complete list of titles in this series, visit the
AMS Bookstore at **www.ams.org/bookstore/**.

CONM 581

Harmonic Analysis and PDEs • Nahmod et al., Editors

This volume is based on the AMS Special Session on Harmonic Analysis and Partial Differential Equations and the AMS Special Session on Nonlinear Analysis of Partial Differential Equations, both held March 12–13, 2011, at Georgia Southern University, Statesboro, Georgia, as well as the JAMI Conference on Analysis of PDEs, held March 21–25, 2011, at Johns Hopkins University, Baltimore, Maryland. These conferences all concentrated on problems of current interest in harmonic analysis and PDE, with emphasis on the interaction between them.

This volume consists of invited expositions as well as research papers that address prospects of the recent significant development in the field of analysis and PDE. The central topics mainly focused on using Fourier, spectral and geometrical methods to treat well-posedness, scattering and stability problems in PDE, including dispersive type evolution equations, higher-order systems and Sobolev spaces theory that arise in aspects of mathematical physics.

The study of all these problems involves state-of-the-art techniques and approaches that have been used and developed in the last decade. The interrelationship between the theory and the tools reflects the richness and deep connections between various subjects in both classical and modern analysis.

ISBN 978-0-8218-6921-5

9 780821 869215

CONM/581

AMS *on the* Web
www.ams.org

AMS