

CONTEMPORARY MATHEMATICS

592

Noncommutative Birational Geometry, Representations and Combinatorics

AMS Special Session on
Noncommutative Birational Geometry,
Representations and Cluster Algebras
January 6–7, 2012
Boston, MA

Arkady Berenstein
Vladimir Retakh
Editors



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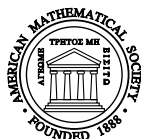
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American Mathematical Society
Providence, Rhode Island

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Preface

The articles in this volume are based on talks given at the Special Session on “Noncommutative birational geometry, representations and cluster algebras” during the Annual Meeting of the American Mathematical Society held in January of 2012 in Boston. The papers presented here deal with various aspects of noncommutative birational geometry and related topics. The focus of the papers is mostly on structure and representations of quantum groups and algebras, braided algebras, rational series in free groups, Poisson brackets on free algebras, and related problems in combinatorics.

The volume is useful for researchers and graduate students in mathematics and mathematical physics who want to be introduced to different areas of current research in the new area of noncommutative algebra and geometry.

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This volume contains the proceedings of the AMS Special Session on Noncommutative Birational Geometry, Representations and Cluster Algebras, held from January 6–7, 2012, in Boston, MA.

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