CONTEMPORARY MATHEMATICS

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Fractal Geometry and Dynamical Systems in Pure and Applied Mathematics I: Fractals in Pure Mathematics

PISRS 2011 International Conference on Analysis, Fractal Geometry, Dynamical Systems and Economics
November 2011: Messina, Sicily, Italy

AMS Special Session on Fractal Geometry in Pure and Applied Mathematics: in Memory of Benoît Mandelbrot
January 2012: Boston, Massachusetts

AMS Special Session on Geometry and Analysis on Fractal Spaces
March 2012: Honolulu, Hawaii

David Carfì
Michel L. Lapidus
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Preface

The Contemporary Mathematics volume

> *Fractal Geometry and Dynamical Systems in Pure and Applied Mathematics* I: *Fractals in Pure Mathematics*

contains papers from talks given at three conferences in 2011–2012, following the passing of Benoît Mandelbrot (widely regarded as the father of fractal geometry) in October of 2010. These meetings are described in chronological order below.

On the occasion of the 2011 Anassilaos International Research Prize in Mathematics, awarded to Michel L. Lapidus (University of California, Riverside), the Permanent International Session of Research Seminars (PISRS) held its first International Meeting

> PISRS 2011: Analysis, Fractal Geometry, Dynamical Systems and Economics.

The conference was held on November 8–12, 2011, at the University of Messina in Sicily, Italy, and was attended by experts in the fields of Fractal Geometry, Dynamical Systems, Number Theory, Noncommutative Geometry, Mathematical and Theoretical Physics, as well as Economics. In addition to approximately 40 experienced researchers participating, the conference included more than 150 students, professors and experts following and attending the meeting. The Award Ceremony for Michel Lapidus took place in Reggio Calabria on Saturday, November 12. The Scientific Committee of PISRS includes over 50 professors and scholars from more than 25 outstanding universities around the world. It has several branches, including Applied Functional Analysis; Biomathematics; Decision and Game Theory; Differential, Fractal and Noncommutative Geometry; Mathematical Methods of Economics, Finance and Quantum Mechanics; Mathematical Physics and Dynamical Systems. The Chairman of PISRS is David Carfi.

The 2012 AMS/MAA/SIAM Joint Mathematics National Meeting, held in Boston in January 2012, included an AMS Special Session on "Fractal Geometry in Pure and Applied Mathematics" in memory of Benoît Mandelbrot. Its organizers were Michel Lapidus, Erin Pearse and Machiel van Frankenhuijsen. In five sessions (including sessions comprised of primarily applied topics), researchers from around the world presented their work in various areas of fractal mathematics. An entire session was devoted to the applications to Physics, Biology, Engineering and Computer Science. During one of the breaks, an experiment was performed which demonstrated the capabilities of fractal antennas. Many speakers described ways in which their work was influenced by the work of Benoît Mandelbrot, and a

special dinner was organized in his honor. Several talks were attended by Aliette Mandelbrot, Benoît's widow, who also gave a short but touching speech.

The Spring 2012 Meeting of the AMS Western Section, held in Honolulu, Hawaii, at the University of Hawaii at Manoa, included a Special Session on "Geometry and Analysis on Fractal Spaces". Its organizers were Michel Lapidus, Lũ' Hùng, John Rock and Machiel van Frankenhuijsen. In four sessions, researchers from around the world presented their work in various areas of fractal mathematics.

This is a collection of papers on fractal geometry (and some aspects of dynamical systems) in pure mathematics. It features articles discussing a variety of connections between these subjects and other fields of mathematics, including probability theory, number theory, geometric measure theory, partial differential equations, global analysis on nonsmooth spaces, harmonic analysis and spectral geometry.

These proceedings were conceived as a means of collecting some of the most recent developments in this active area of research, and also to bring together several survey and research expository articles, as a means of introducing new researchers and graduate students to the forefront of the field. The present volume focuses on the more mathematical aspects of the field. Its companion volume, entitled *Fractal Geometry and Dynamical Systems in Pure and Applied Mathematics II* and subtitled *Fractals in Applied Mathematics,* focuses on more applied topics, including the applications of fractal geometry and dynamical systems to other sciences.

David Carfi,
Michel L. Lapidus,
Erin P. J. Pearse, and
Machiel van Frankenhuijsen.

March 2013

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Articles in this volume cover fractal geometry (and some aspects of dynamical systems) in pure mathematics. Also included are articles discussing a variety of connections of fractal geometry with other fields of mathematics, including probability theory, number theory, geometric measure theory, partial differential equations, global analysis on non-smooth spaces, harmonic analysis and spectral geometry.

The companion volume (Contemporary Mathematics, Volume 601) focuses on applications of fractal geometry and dynamical systems to other sciences, including physics, engineering, computer science, economics, and finance.

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