CONTEMPORARY MATHEMATICS

606

Centre de Recherches Mathématiques Proceedings

Women in Numbers 2: Research Directions in Number Theory

BIRS Workshop WIN2—Women in Numbers 2 November 6–11, 2011 Banff International Research Station Banff, Alberta, Canada

> Chantal David Matilde Lalín Michelle Manes Editors



American Mathematical Society Providence, Rhode Island

Centre de Recherches Mathématiques Montréal, Québec, Canada Women in Numbers 2: Research Directions in Number Theory



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Preface

The second Women In Numbers workshop (WIN2) was held November 6– 11, 2011, at the Banff International Research Station (BIRS) in Banff, Alberta, Canada. The workshop was one of several Research Collaboration Conferences for Women (http://research.microsoft.com/en-us/projects/rccw/) held in recent years. These conferences bring together senior women in a particular mathematical research area—in this case number theory—to lead research projects during a weeklong workshop. The participants in the project groups are women graduate students, postdocs, and junior faculty. The benefit of these workshops is in both directions: senior women meet, mentor, and collaborate with young researchers; junior women encounter important new research problems and develop a network of colleagues, supporters, and mentors.

Forty-one mathematicians attended the WIN2 workshop, which was organized by the three editors of this volume. The organizers first invited senior researchers to lead the projects, and then assigned the other participants to projects according to their expertise and interests. The result was eight working groups of four to six members each, including two leaders per group.

This volume contains survey articles written by leaders of the WIN2 working groups, reports on new research conducted by the working groups at the conference, and additional articles submitted by researchers who are part of the Women in Numbers Network (http://womeninnumbertheory.org/).

Workshop project titles and lectures

WIN2 was a working conference, with several hours each day devoted to research in project groups. In addition, there were a few talks each day. During the first three days, group leaders introduced their general areas of research and proposed their projects. On the final two days, group members described their progress and shared their plans to complete the work.

- Elliptic surfaces and Mahler measure
 - Lectures: Marie-José Bertin, Matilde Lalín
 - Group members: Amy Feaver, Jenny Fuselier, Michelle Manes
- Analytic number theory
 - Lectures: Chantal David, Heekyoung Hahn
 - Group members: Shabnam Akhtari, Min Lee, Lola Thompson
- Number theory in functions fields and algebraic geometry over finite fields
 - Lectures: Alina Bucur, Melanie Matchett Wood
 - Group members: Jing Hoelscher, Renate Scheidler
- Arithmetic algebraic geometry
 - Lectures: Alina Cojocaru, Alice Silverberg

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- Group members: Rachel Davis, Antonella Perucca, Katherine Stange, Diane Yap
- $\bullet\,$ K-theory and algebraic number theory
 - Lectures: Wieslawa Niziol, Sujatha Ramdorai
 - Group members: Veronica Ertl, Bregje Pauwels, Ila Varma
- Arithmetic geometry
 - Lectures: Rachel Pries, June Zhu
 - Group members: Rebecca Bellovin, Sharon Garthwaite, Ekin Ozman, Cassie Williams
- Modular forms
 - Lectures: Ling Long, Gabriele Nebe
 - Group members: Sarah Chisholm, Alyson Deines, Holly Swisher
- Arithmetic intersection theory
 - Lectures: Kristin Lauter, Bianca Viray
 - Group members: Jackie Anderson, Jennifer Balakrishnan, Jennifer Park

Contributions to this volume

The editors requested one or two contributions from each working group at the WIN2 workshop. In addition, we solicited articles through the Women in Numbers Network (mailing list and web site). All submissions to this volume were sent to anonymous referees, who assessed them as correct and worthwhile contributions to these proceedings.

Workshop website

https://www.birs.ca/events/2011/5-day-workshops/11w5075

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- Banff International Research Station
- $\bullet\,$ Microsoft Research
- Pacific Institute for the Mathematical Sciences
- The Number Theory Foundation

We would like to thank the referees whose careful and dedicated work have been crucial in assuring the quality of this publication.

> July 2013 Chantal David, Concordia University, Canada Matilde Lalín, Université de Montréal, Canada Michelle Manes, University of Hawai'i at Mānoa, USA

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The Centre de Recherches Mathématiques (CRM) was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM receives funding from the Natural Sciences and Engineering Research Council (Canada), the FRQNT (Québec), the NSF (USA), and its partner universities (Université de Montréal, McGill, UQAM, Concordia, Université Laval, Université de Sherbrooke and University of Ottawa).

The second Women in Numbers workshop (WIN2) was held November 6–11, 2011, at the Banff International Research Station (BIRS) in Banff, Alberta, Canada. During the workshop, group leaders presented open problems in various areas of number theory, and working groups tackled those problems in collaborations begun at the workshop and continuing long after.

This volume collects articles written by participants of WIN2. Survey papers written by project leaders are designed to introduce areas of active research in number theory to advanced graduate students and recent PhDs. Original research articles by the project groups detail their work on the open problems tackled during and after WIN2. Other articles in this volume contain new research on related topics by women number theorists.

The articles collected here encompass a wide range of topics in number theory including Galois representations, the Tamagawa number conjecture, arithmetic intersection formulas, Mahler measures, Newton polygons, the Dwork family, elliptic curves, cryptography, and supercongruences.

WIN2 and this Proceedings volume are part of the Women in Numbers network, aimed at increasing the visibility of women researchers' contributions to number theory and at increasing the participation of women mathematicians in number theory and related fields.

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