

# CONTEMPORARY MATHEMATICS

614

## Automorphic Forms and Related Geometry: Assessing the Legacy of I.I. Piatetski-Shapiro

Conference on Automorphic Forms and Related Geometry:  
Assessing the Legacy of I.I. Piatetski-Shapiro  
April 23–27, 2012  
Yale University, New Haven, CT

James W. Cogdell  
Freydoon Shahidi  
David Soudry  
Editors



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## Preface

Ilya I. Piatetski-Shapiro, who passed away on 21 February 2009, was a leading figure in the theory of automorphic forms. From April 23–27, 2012 the conference *Automorphic Forms and Geometry: Assessing the Legacy of I.I. Piatetski-Shapiro* was held at Yale University to assess the legacy of his work. The organizing committee for the conference consisted of J. Cogdell, S. Gelbart, R. Howe, D. Jiang, P. Sarnak, F. Shahidi, and D. Soudry.

The conference attempted both to summarize and consolidate the progress that was made during Piatetski-Shapiro's lifetime by him and his substantial group of co-workers, and to promote future work by identifying fruitful directions of further investigation. It was organized around several themes that reflected Piatetski-Shapiro's main foci of work and that have promise for future development. In each area, there were talks to review the current state of affairs with special attention to Piatetski-Shapiro's contributions, and other talks to report on current work and to outline promising avenues for continued progress. The themes selected were: functoriality and converse theorems, local and global  $L$ -functions and their periods,  $p$ -adic  $L$ -functions and arithmetic geometry, complex geometry, and analytic number theory.

The contents of this volume reflect most of the talks that were presented at the conference. Not all speakers contributed an article, while some contributors were invited to speak but were unable; their contributions represent the talk they would have given. There are several surveys on the work of Piatetski-Shapiro and mathematics influenced by him. Many contributions represent current research of the authors. All of them represent various aspects of the legacy of Piatetski-Shapiro.

For those interested in further investigation into the legacy of Piatetski-Shapiro, in addition to this volume, we refer you to the festschrift prepared for his sixtieth birthday (*Festschrift in honor of Piatetski-Shapiro on the occasion of his sixtieth birthday*, Israel Math. Conf. Proc., 2 & 3, Weizmann, Jerusalem, 1990); his selected works and the commentary therein, which coincided with his seventieth birthday (*Selected works of Ilya Piatetski-Shapiro*, edited and with commentaries by J. Cogdell, S. Gindikin, P. Sarnak, P. Deligne, S. Gelbart, R. Howe and S. Rallis. American Mathematical Society, Providence, RI, 2000); and the memorial article, which unfortunately coincided with what would have been his eightieth birthday (*Ilya Piatetski-Shapiro, In Memoriam*, coordinating editors J. Cogdell, S. Gelbart, and P. Sarnak, *Notices of the AMS* **57** (2010), 1260–1275).

On behalf of the organizing committee, we would like to thank all of the speakers that contributed to the conference, whether they were able to contribute to this volume or not, those that contributed to this volume even if they could not speak, and all of the many participants who filled Davies Auditorium at Yale. We also



thank those that sponsored the conference: The National Science Foundation, The Number Theory Institute, The Institute for Mathematics and its Applications, The Department of Mathematics of the University of Minnesota, The Department of Mathematics of Yale University, and Yale University. In addition the editors would like to offer their heartfelt thanks to the anonymous referees that helped make this volume possible. But mostly we would like to thank Piatetski-Shapiro for the great legacy that he left us.

J. Cogdell, F. Shahidi, and D. Soudry

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This volume contains the proceedings of the conference Automorphic Forms and Related Geometry: Assessing the Legacy of I.I. Piatetski-Shapiro, held from April 23–27, 2012, at Yale University, New Haven, CT.

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