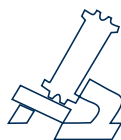# CONTEMPORARY MATHEMATICS

Israel Mathematical Conference Proceedings

Variational and Optimal Control Problems on Unbounded Domains

A Workshop in Memory of Arie Leizarowitz
January 9–12, 2012
Haifa, Israel

Gershon Wolansky
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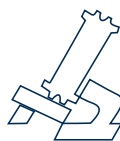Editors

Contemporary Mathematics

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Contents

Preface

A workshop "Variational and optimal control problems on unbounded domains," in memory of our late colleague Professor Arie Leizarowitz, took place at the Technion in Haifa, Israel, on January 9–12, 2012, under the auspices of the Israel Science Foundation (ISF) and the Technion Center for Mathematical Sciences. The workshop brought together a selected group of experts in optimal control theory and of the calculus of variations working on problems on unbounded domains. Altogether, over 35 participants from 11 countries attended the Workshop.

These proceedings are the tangible record of the Workshop. Most of the papers collected here have been contributed by participants in the Workshop. In some cases, they have chosen to submit manuscripts which depart from the texts of their lectures. All submissions have been carefully refereed.

We acknowledge with thanks the support provided for the Workshop by the Israel Science Foundation (ISF) and the Technion Center for Mathematical Sciences (CMS).

The Editors

Biography and Bibliography of Arie Leizarowitz

Arie Leizarowitz (1953–2010) was a Professor of Mathematics at the Technion-Israel Institute of Technology. He was born in Tel Aviv in 1953. After obtaining his B.Sc degree in Physics from Tel Aviv University, Arie Leizarowitz went on to serve as an officer at the Israeli Air Force. He received his M.Sc. degree in Applied Mathematics at Tel Aviv University in 1980 and his Ph.D. in Mathematics from the Weizmann Institute under the guidance of Professor Zvi Artstein in 1984. Arie Leizarowitz used his prestigious Weizmann Fellowship to spend two years at the Institute for Mathematics and its Applications at the University of Minnesota. This was followed by a two year appointment as a Zeev Nehari Assistant Professor at Carnegie Mellon University. In 1988 he joined the Technion. Over his professional career, Professor Leizarowitz published about 60 research papers in leading international journals and a widely-recognized monograph with A. Haurie and D. A. Carlson (Springer, 1991). His main contributions were in infinite horizon optimal control theory, where he developed a number of fundamental, highly recognized results on the existence of solutions to infinite horizon optimization problems including discrete and continuous time problems, and deterministic and stochastic problems.

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Workshop Program

January 9–12, 2012

Monday, January 9

09:00–09:40   Opening

09:45–10:25   Terry Rockafellar
Applications of variational analysis to the stability and control of economic equilibrium

10:30–11:10   Alexander Ioffe
Optimal control of differential inclusions: results and open problems

11:30–12:10   Arkadii Kryazhimskii
On the solvability of closed-loop control problems for partially observable systems

12:10–12:50   Rafal Goebel
Stabilization problems and convex solutions to stationary Hamilton-Jacobi equations

14:40–15:20   Sergey Aseev
Finite-horizon approximations and the Pontryagin maximum principle for a class of infinite-horizon optimal control problems

15:25–16:05   Vladimir Veliov
Maximum principle for infinite-horizon optimal control problems with dominating discount

16:25–17:05   Alberto Seeger
Spectral techniques for the analysis of differential inclusions

17:10–17:50   Geraldo Nunes Silva
Generalized invexity in infinite horizon optimal control problem

Tuesday, January 10

09:00–09:40   Paul Rabinowitz
              On an Allen-Cahn model equation

09:45–10:25   Albert Fathi
              *Convergence of discounted solutions of the Hamilton-Jacobi
              equation*

10:30–11:10   Moshe Marcus
              *Infinite horizon variational problems related to a fourth order
              o.d.e.*

11:30–12:10   Alexander Zaslavski
              Turnpike properties in the calculus of variations

12:10–12:50   Pierre Cartigny
              *Explicit solutions for singular infinite horizon calculus of
              variations problems*

14:40–15:20   Avi Berman
              On the second eigenvalues of matrices associated with TCP

15:25–16:05   Sabine Pickenhain
              *Infinite horizon optimal control problems - a Hilbert space
              approach*

16:25–17:05   Marshall Slemrod
              Entropy, elasticity, and embedding

17:10–17:50   Jacob Kogan
              Feature selection over distributed data streams

Wednesday, January 11

09:00–09:40   Uriel Rothblum
Bandit problems and linear algebra

09:45–10:25   Vivek Borkar
Relative value iteration for controlled diffusions

10:30–11:10   Adam Shwartz
Approximations in MDPs

11:30–12:10   Igor Evstigneev
Von Neumann-Gale dynamical systems with applications in finance

12:10–12:50   Vladimir Rotar
On pathwise asymptotic optimality in dynamic stochastic control problems

14:40–15:20   Rami Atar
A differential game for a resource allocation problem

15:25–16:05   George Weiss
The optimal control of wave energy converters

16:25–17:05   Ezra Zeheb
What is preferable: positive or negative, real or imaginary?

17:10–17:50   Valery Glizer
Singular solution of an infinite horizon linear-quadratic optimal control problem with state delays

Thursday, January 12

09:00–09:40   Zvi Artstein
              Infinite horizon optimization and Young measures

09:45–10:25   Vladimir Gaitsgory
              *Averaging and linear programming approaches to singularly
              perturbed optimal control problems*

10:45–11:25   Yacov Tsur
              Resource management in a stochastic and fragile environment

11:30–12:10   Michael Margaliot
              *Stability analysis of positive linear switched systems:
              a variational approach*

14:30–15:10   Marc Quincampoix
              Optimal control and linear programming analysis

15:10–15:50   Nikolai Osmolovskii
              On sufficient second-order conditions in optimal control

15:15–15:55   Gershon Wolansky
              Limit theorems for optimal mass transportation

15:55–16:35   Ilya Ioslovich
              On optimal irrigation scheduling

16:35–16:45   Closing remarks

List of Participants

Zvi Artstein
The Weizmann Institute of Science,
Israel

Sergey Aseev
International Institute for Applied
Systems Analysis, Austria

Rami Atar
Technion, Israel

Avi Berman
Technion, Israel

Vivek S. Borkar
Indian Institute of Technology Bombay,
India

Pierre Cartigny
Greqam-Université de la M'editerranée,
France

Igor Evstigneev
University of Manchester, Great Britian

Albert Fathi
ENS-Lyon and Institut Universitaire de
France, France

Vladimir Gaitsgory
University of South Australia, Australia

Valery Y. Glizer
ORT Braude College, Israel

Rafal Goebel
Loyola University Chicago, USA

Alexander Ioffe
Technion, Israel

Ilya Ioslovich
Technion, Israel

Jacob Kogan
University of Maryland Baltimore
County, USA

Arkadii Kryazhimskii
International Institute for Applied
Systems Analysis, Austria and Steklov
Mathematical Institute, Russian
Academy of Sciences, Russia

Elena Litsyn
Ben-Gurion University of the Negev,
Israel

Moshe Marcus
Technion, Israel

Michael Margaliot
Tel Aviv University, Israel

Geraldo Nunes Silva
TUNESP - Universidade Estadual
Paulista, Brazil

Nikolai P. Osmolovskii
Systems Research Institute, Warszawa,
Poland

Sabine Pickenhain
Cottbus University of Technology,
Germany

Marc Quincampoix
University of Brest, France

Paul Rabinowitz
University of Wisconsin-Madison, USA

R. Tyrrell Rockafellar
University of Washington, USA

Vladimir Rotar
San Diego State University, USA

Uriel Rothblum
Technion, Israel

Alberto Seeger
University of Avignon, France

Adam Shwartz
Technion, Israel

Marshall Slemrod
University of Wisconsin, Madison, USA
and Weizmann Institute of Science,
Israel

Yacov Tsur
The Hebrew University of Jerusalem,
Israel

Vladimir Veliov
Vienna University of Technology,
Austria

George Weiss
Tel Aviv University, Israel

Gershon Wolansky
Technion, Israel

Alexander J. Zaslavski
Technion, Israel

Ezra Zeheb
Technion and Jerusalem College
of Engineering, Israel

Variational and Optimal Control Problems on Unbounded Domains   •   Wolansky et al., Editors

This volume contains the proceedings of the workshop on Variational and Optimal Control Problems on Unbounded Domains, held in memory of Arie Leizarowitz, from January 9–12, 2012, in Haifa, Israel.

The workshop brought together a select group of worldwide experts in optimal control theory and the calculus of variations, working on problems on unbounded domains.

The papers in this volume cover many different areas of optimal control and its applications. Topics include needle variations in infinite-horizon optimal control, Lyapunov stability with some extensions, small noise large time asymptotics for the normalized Feynman-Kac semigroup, linear-quadratic optimal control problems with state delays, time-optimal control of wafer stage positioning, second order optimality conditions in optimal control, state and time transformations of infinite horizon problems, turnpike properties of dynamic zero-sum games, and an infinite-horizon variational problem on an infinite strip.

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