

CONTEMPORARY MATHEMATICS

620

Algebraic Topology: Applications and New Directions

Stanford Symposium on
Algebraic Topology: Applications and New Directions
July 23–27, 2012
Stanford University, Stanford, CA

Ulrike Tillmann
Søren Galatius
Dev Sinha
Editors



American Mathematical Society

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In honour of Gunnar Carlsson, Ralph Cohen, and Ib Madsen

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Preface

The Stanford Symposium “Algebraic Topology: Applications and New Directions” took place 23–27 July 2012. The conference was held in honour of Gunnar Carlsson, Ralph Cohen and Ib Madsen, who celebrated their 60th and 70th birthdays that year. It showcased current research in Algebraic Topology reflecting the celebrants’ broad interests and profound influence on the subject. The topics varied broadly from stable equivariant homotopy theory to persistent homology and application in data analysis, covering topological aspects of quantum physics such as string topology and geometric quantisation, examining homology stability in algebraic and geometric contexts, including algebraic K-theory and the theory of operads.

There were 250 participants attending the conference and 50 talks were delivered in plenary and parallel sessions. A special session was devoted to applied algebraic topology, and a lively panel discussion explored the theme of “Future directions in algebraic topology”.

Speakers were invited to contribute to these proceedings. As editors we were keen to catch the vibrancy and diversity of the subject area as evident at the symposium and produce a volume that would honour the three celebrants. Authors were invited to submit research and survey articles as well as work that is forward looking and speculative.

The conference was supported financially primarily by the National Science Foundation and the Stanford University Mathematics Research Center. The scientific programme was much enhanced by many social events including a barbecue, a hike and a conference dinner. As organisers of the symposium we would like to express our deep gratitude to Ralph Cohen and all those who assisted him for the organisation of these and the generous hospitality offered throughout the week. Finally, we would like to thank all speakers and participants for their excellent contributions and enthusiasm.

Ulrike Tillmann
Søren Galatius
Dev Sinha

November 2013

Scientific Programme

Plenary Lectures

Andrew Blumberg

New directions for trace methods

Bill Dwyer

Operads and higher knots

Dan Freed

3d TQFTs through the lens of the cobordism hypothesis

Søren Galatius

Homology of moduli spaces of high dimensional manifolds

Lars Hesselholt

Real algebraic K-theory

Michael Hopkins

Equivariant multiplicative closure

Matthew Kahle

Topology of random flag simplicial complexes

Jacob Lurie

p -Divisible groups, and character theory

Graeme Segal

Semi-infinite homotopy theory and noncommutative geometry

Ulrike Tillmann

On the work of three eminent topologists

Craig Westerland

A higher chromatic analogue of the image of J

Parallel Sessions

Alejandro Adem

Equivariant K-theory and spaces of commuting elements in a compact Lie group

David Ayala

Higher categories are sheaves on manifolds

Clark Barwick

Dévissage

Alexander Berglund

Homological stability for automorphisms of manifolds

Julie Bergner

Diagrams encoding group actions

Carl-Friedrich Bökigheimer

Homology operations for moduli spaces

Tom Church

Homological stability via Koszul duality for FI-modules

Chris Douglas

Fusion categories and field theories

Dan Dugger

Motivic characteristic classes for quadratic bundles

Bjorn Ian Dundas

Higher topological Hochschild homology

Nora Ganter

Elliptic Schubert calculus

Teena Gerhardt

Algebraic K-theory and Witt vectors

Boris Goldfarb

On the algebraic structure of geometric group rings

Jesper Grodal

F-isomorphism in group cohomology implies isomorphism

Ian Hambleton

Co-compact discrete group actions and the assembly map

Allen Hatcher

Stable homology of spaces of graphs

Richard Hepworth

String topology of classifying spaces

- Kathryn Hess
The divided powers functor on symmetric sequences
- Mike Hill
Equivariant Symmetric Monoidal Categories
- Dan Isaksen
*From motivic to classical homotopy theory:
Reverse engineering the classical Adams-Novikov spectral sequence*
- Nitu Kitchloo
The stable symplectic category and geometric quantization
- John Klein
*On the quantization of fluctuating currents:
an application of algebraic topology to statistical mechanics*
- Ernesto Lupercio
Non-commutative toric varieties
- Mike Mandell
Localization sequences in THH
- Paul Norbury
Gromov-Witten invariants of the two-sphere and mirror symmetry
- Kate Poirier
Compactifying string topology
- Dan Ramras
Stable representation theory and the geometry of flat connections
- Oscar Randal-Williams
Homological stability for moduli spaces of manifolds
- Paolo Salvatore
*Cellular decompositions of planar configuration spaces
and the Fulton Mac Pherson operad*
- Dev Sinha
Cohomology of symmetric and alternating groups
- Nathalie Wahl
Universal operations in Hochschild homology
- Michael Weiss
Smooth maps to the plane and Pontryagin classes
- Kirsten Wickelgren
Investigating the section conjecture

Special Session: Applied Topology

Henry Adams

Evasion paths in mobile sensor networks

Justin Curry

Cosheaves and dualities in generalized sensor networks

Daniel Müllner

Consistent scale selection for exploratory visualization and analysis of data sets

Monica Nicolau

*Unraveling the biology of disease
through data transformations and topological data analysis*

Primož Skraba

Persistence of Random Points

Mikael Vejdemo-Johansson

*Computation of spectral sequences of double complexes,
with applications to persistent homology*

Panel Discussion: Future Directions of Algebraic Topology

Panelists:

Alejandro Adem (moderator)

David Ayala

Bill Dwyer

Dan Freed

Kathryn Hess

Michael Hopkins

Nick Kuhn

List of Participants and Conference Photo

Henry Adams, Alejandro Adem, Mohammed Alkadhi, Mio Alter, Andres Angel, Miguel Angel, Benjamin Antieau, Omar Antolin Camarena, Peter Arndt, Maia Averett, David Ayala, Emel Aydin, Nils Baas, Kerstin Baer, Jeffrey Bailes, Anthony Bak, Luis Torres, Tarje Bargheer, Tobias Barthel, Clark Barwick, Maria Basterra, Marzieh Bayeh, Mahmoud Benkhalifa, Alexander Berglund, Julie Bergner, Ahmet Beyaz, Andrew Blumberg, Carl-Friedrich Bödigheimer, Anna Marie Bohmann, Tyler Borrman, Boris Botvinnik, James Buban, Jonathan Campbell, Jose Cantarero, Gunnar Carlsson, Raymond Cavalcante, Man Chuen Cheng, Eungchun Cho, Dan Christensen, Thomas Church, Ralph Cohen, Xiaoyi Cui, Justin Curry, Sandip Deb, Rafael Diaz, Christopher Douglas, Diego Daniel Duarte Vogel, Daniel Dugger, Bjorn Ian Dundas, William Dwyer, Daniela Egas Santander, Emmanuel Ekwedike, Ernest Fontes, Dan Freed, Greg Friedman, Søren Galatius, Nora Ganter, Andres Garcia, Charles Garwin, Josh Genauer, David Gepner, Teena Gerhardt, Robert Ghrist, Chad Giusti, Boris Goldfarb, Mauricio Gomez, Antonio Gomez-Tato, Jesper Grodal, Moritz Groth, Casper Guldberg, Philip Hackney, Nomana Intekhab Hadi, Ian Hambleton, Robert Hank, John Harper, Allen Hatcher, Rune Haugseng, Drew Heard, Elizabeth Henning, Richard Hepworth, Kathryn Hess, Lars Hesselholt, Galo Higuera Rojo, Michael Hill, Joseph Hirsh, Michael Hopkins, Jonathan Huang, Thomas Huettemann, Johannes Huisman, Thomas Hunter, David Hurtubise, Brian Hwang, Kevin Iga, Michele Intermont, Dan Isaksen, Daniel Ishak, Etienne Jacques, Xiaoguang Jiang, John Jones, Matthew Kahle, Sara Kalisnik, Sadok Kallel, Mohammad Javad Karimi Abadchi, Max Karoubi, Ryo Kato, Nitu Kitchloo, John Klein, Jennifer Kloke, Johan Konter, Mahdi Kouretchian, Robin Koytcheff, Nick Kuhn, Dileep Kumar, Pankaj Kumar, Aradhana Kumari, Alexander Kupers, Amuzu Kwame, Anssi Lahtinen, Jonathan Lee, Fabian Lenhardt, Ji Li, John Lind, Ayelet Lindenstrauss, Lydia Liu, Ernesto Lupercio, Jacob Lurie, Ib Madsen, Cary Malkiewich, Eric Malm, Ashis Mandal, Michael Mandell, Michael Martinez, Takuo Matsuoka, Kyle Matthews, Justin Mauger, Daniel Maya, Joaquin Maya Duque, Kristen Mazur, John McCleary, Sebastian Meinert, Mona Merling, Haynes Miller, Jeremy Miller, Steve Mitchell, Kristian Jonsson Moi, Syunji Moriya, Syunji Moriya, Dmitriy Morozov, John Mosley, Daniel Müllner, Tommy Murphy, Hirofumi Nakai, Mara Neusel, Nicholas Nguyen, Monica Nicolau, Zhaohu Nie, Joao Miguel Nogueira, Sam Nolen, Paul Norbury, Kyle Ormsby, Angelica Osorno, Martin Palmer, Matthew Pancia, John Pardon, Arthur Parzygnat, Jose Perea, Nathan Perlmutter, Alexander Perry, Kate Poirier, Kate Ponto, Hamidreza Rahimi, Priyanka Rajan, Eliharintsoa Rajaonarimirana, Alain Patrick Rajaonari-son, Daniel Ramras, Oscar Randal-Williams, Erika Refsland, Emily Riehl, Manuel Rivera, Aaron Royer, Christian Rüschoff, Hal Sadofsky, Rustam Sadykov, Kadriye

Nur Saglam, Edward Salamanca, David Sallach, Renato Salmeron, Paolo Salvatore, Marc Sanders, Beren Sanders, Joao Santos, Jenny Santoso, Shyam Sarkar, Justin Scarfy, Patrick Schultz, Paul Sebexen, Graeme Segal, Carlos Segovia, Debasis Sen, Jay Shah, Brooke Shipley, Kyler Siegel, Alexis Sien, Dev Sinha, Primoz Skraba, Jacek Skryzalin, Peter Smillie, Jeffrey Smith, Yongjin Song, David Spivak, David Sprehn, Don Stanley, Nathaniel Stapleton, Marc Stephan, Maxim Stykow, Bishnu Hari Subedi, Hiro Lee Tanaka, Tara Taylor, Courtney Thatcher, Marius Thaumle, Ulrike Tillmann, Sean Tilson, Takeshi Torii, TriThang Tran, Arnav Tripathy, Mark Ullmann, Son Van, Marco Varisco, Maria Elena Vazquez, Mikael Vejdemo-Johansson, Chaman Singh Verma, Deborah Vicinsky, Darryl Wade, Nathalie Wahl, Gaohong Wang, Ben Ward, Michael Weiss, Craig Westerland, Kirsten Wickelgren, Ben Williams, Stephen Wilson, Enxin Wu, Miguel Xicotencatl, Zhouli Xu, Andrew Yarmola, Carolyn Yarnall, Inna Zakharevich, Marcus Zibrowius, Yan Zou.



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