CONTEMPORARY MATHEMATICS

623

Recent Advances in Representation Theory, Quantum Groups, Algebraic Geometry, and Related Topics

AMS Special Sessions on
Geometric and Algebraic Aspects of Representation Theory and
Quantum Groups and Noncommutative Algebraic Geometry
October 13–14, 2012
Tulane University, New Orleans, LA

Pramod N. Achar
Dijana Jakelić
Kailash C. Misra
Milen Yakimov
Editors

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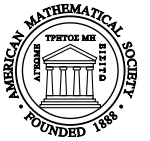

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Preface

Representation theory, along with its interactions with other areas of mathematics, such as noncommutative algebra, algebraic geometry, and mathematical physics, constitutes a major area of current mathematical research. Many aspects of the traditional theory of finite-dimensional semisimple Lie algebras, due to Killing, Cartan, Weyl, and others around the turn of the twentieth century, have found vast generalizations in recent decades in a number of different directions such as Kac–Moody algebras (especially affine Lie algebras), vertex algebras, and "quantum" and "super" versions of these algebras. In the case of quantum algebras in particular, classical Lie theory makes contact both with the abstract study of Hopf algebras and general techniques from noncommutative ring theory. Quantum analogues of varieties such as affine spaces, tori, and Schubert cells play a key role here. This brings us to connections with ordinary (commutative) geometry. A number of major advances in representation theory in recent decades have been achieved by translating algebraic questions into terms involving either algebraic topology (via perverse sheaves or $\mathcal{D}$-modules) or algebraic geometry (via coherent sheaves and K-theory) of flag varieties, quiver varieties, and related spaces.

At the Fall Southeastern Section Meeting of the American Mathematical Society in New Orleans, LA, during October 13–14, 2012, two special sessions on: "Geometric and Algebraic Aspects of Representation Theory" and "Quantum Groups and Noncommutative Algebraic Geometry," respectively, were organized by the editors. These were mathematically robust and engaging sessions in which a number of senior and junior researchers presented their work on a variety of research topics. Several of the speakers of these two special sessions have contributed to this volume. We believe that the developments brought together in this volume constitute a valuable contribution to the literature in representation theory. The diversity of topics covered in this volume will help readers working in one branch of representation theory to discover connections to other branches of which they may not have otherwise been aware. This volume is a useful reference to both established mathematicians and junior researchers in representation theory.

We would like to thank all the speakers at these two special sessions, especially those who have contributed to this proceedings volume. We are grateful to all the referees for their thoughtful and timely reviews. We would also like to thank Edward Dunne for suggesting that we put this proceedings together, and Christine Thivierge for her guidance and help through the editorial process.

The Editors

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Brian Parshall (University of Virginia), *Shifted generic cohomology.*

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Susan Montgomery (University of Southern California), *Computing Frobenius-Schur indicators for doubles of groups.*

Siu-Hung Ng (Iowa State University), *Total indicators of the representations of quasi-Hopf algebras.*

Anne Schilling (University of California, Davis), *A uniform combinatorial model for Kirillov-Reshetikhin crystals and specialized Macdonald polynomials.*

Chelsea Walton (Massachusetts Institute of Technology), *Quantum binary polyhedral groups and their actions on quantum planes.*

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This volume contains the proceedings of two AMS Special Sessions "Geometric and Algebraic Aspects of Representation Theory" and "Quantum Groups and Noncommutative Algebraic Geometry" held October 13–14, 2012, at Tulane University, New Orleans, Louisiana.

Included in this volume are original research and some survey articles on various aspects of representations of algebras including Kac–Moody algebras, Lie superalgebras, quantum groups, toroidal algebras, Leibniz algebras and their connections with other areas of mathematics and mathematical physics.

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