CONTEMPORARY MATHEMATICS

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The Mathematics of Decisions, Elections, and Games

AMS Special Sessions on
The Mathematics of Decisions, Elections, and Games
January 4, 2012, Boston, MA
January 11–12, 2013, San Diego, CA

Karl-Dieter Crisman
Michael A. Jones
Editors

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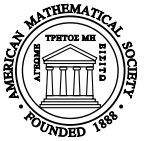

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Contents

Preface

The majority of papers in this collection accompanied talks from the AMS Special Sessions on the Mathematics of Decisions, Elections, and Games from the 2012 and 2013 Joint Mathematics Meetings of the AMS and MAA. These sessions were organized by Karl-Dieter Crisman (Gordon College), Michael A. Jones (Mathematical Reviews), and Michael Orrison (Harvey Mudd College). The one exception is the paper based on a talk from the AMS Special Session on The Redistricting Problem; this session was organized by Daniel Goroff (Harvey Mudd College) and Daniel Ullman (George Washington University) for the 2009 Joint Mathematics Meetings. The full programs for the 2012 and 2013 AMS Special Sessions on the Mathematics of Decisions, Elections, and Games can be found online by searching jointmathematicsmeetings.org.

Decision theory, voting theory, and game theory are three intertwined areas of mathematics that involve making optimal decisions under different contexts. Although these areas consist of their own mathematical results, much of the recent research in these areas involve developing and applying new perspectives from their intersection with other branches of mathematics, such as algebra, representation theory, combinatorics, convex geometry, dynamical systems, etc. The papers in this volume highlight and exploit the mathematical structure of decisions, elections, and games to model and to analyze problems from the social sciences.

In what follows we give a short overview of the papers in this collection. To those new to the area, we wish to emphasize that many different types of mathematics can be profitably used in this interdisciplinary context.

Both *Redistricting and District Compactness* by Corcoran and Saxe and *Fair Division and Redistricting* by Landau and Su focus on the redistricting problem: carving up a state into congressional districts. The former discusses different measures used to evaluate proposed districting plans. These measures take into account the geometric shape of both the state and the districts as well as the spread of the population throughout the district. The latter paper offers a different perspective on redistricting by viewing it as a fair division or cake-cutting problem in which the state is viewed as a cake to be divided into districts that are allocated to one of two political parties.

The next two papers fall under the topic of judgment aggregation, which may be viewed as being at the intersection of social choice theory and decision theory. While social choice theory is concerned with aggregating preferences of voters to arrive at a societal ranking, judgment aggregation is a burgeoning area focusing on aggregating individuals' judgments on interconnected propositions to arrive at a collective judgment.

In *When Does Approval Voting Make the "Right Choices"?*, Brams and Kilgour examine different contexts in which an individual voter approves of a proposal based on the proposal's probability of being right (or good or just) and the voter's probability of making a correct judgment of whether it is right (or wrong). For multiple proposals in which more than one proposal can be approved and under probabilistic settings, they determine conditions for when the most approved proposals have the greatest probability of being right, relating their results to the Condorcet Jury Theorem.

Nehring and Pivato take a more axiomatic approach to judgment aggregation in *How Indeterminate is Sequential Majority Voting? A Judgement Aggregation Perspective*. Similar to other choice problems, the order in which decisions are made can matter. This paper continues a research program exploring in exactly what ways the order in which judgments are made can affect the actual final judgments on various propositions, in this case showing that a very large number of natural examples exhibit various forms of this indeterminacy in sequential majority votes.

Several papers continue a long tradition in the field of using geometry and combinatorial objects to analyze a wide variety of voting questions. Stenson, in *Weighted Voting, Threshold Functions, and Zonotopes*, looks at a generalization of simple games and weighted voting systems. She explains how these games, and their winning/losing coalitions, correspond to vertices and other geometric aspects of particular polytopes and their duals. *The Borda Count, the Kemeny Rule, and the Permutahedron* by Crisman takes the geometric-combinatorial object called the permutahedron, and examines its symmetries with the goal of understanding social preference functions, which return not just winners but (sets of) rankings. The main result uses representation theory to characterize the most symmetric social preference functions as a one-dimensional continuum connecting the two rules in the title.

In *Double-Interval Societies*, Klawe, Nyman, Scott, and Su consider the effect of geometric constraints in a model in which voters approve of positions in a linear spectrum. When each voter's approval set is represented by two disjoint closed intervals and when every pair of voters agree on some position, the authors determine a lower bound for the approval ratio. They also construct societies with low approval ratios by relating the double-interval models to the arrangement of n symbols in which each symbol appears twice, thereby relating the continuous geometry to discrete sequences.

By contrast, *Voting for Committees in Agreeable Societies* by Davis, Orrison, and Su focuses more on combinatorial aspects of agreeability, supposing that voters each approve of a certain size subset of candidates with a goal of selecting a committee of a possibly different size. Using a graph called the Johnson graph, they prove a number of results regarding what proportion of voters will be satisfied with the results under different suppositions about the distribution of voters' intent.

Ratliff's paper *Selecting Diverse Committees with Candidates from Multiple Categories* approaches committees from a different perspective. His focus is on selecting committees where candidates may be in several categories, and where it is desirable to have a committee with members from each of these categories. Using a combination of explicit examples and point-counting arguments with Ehrhart polynomials, both probabilistic and exact results regarding admissible ballot types to obtain these outcomes are derived.

Finally, we have three papers examining game theory from different mathematical perspectives. In *Expanding the Robinson-Goforth System for* 2×2 *Games*, Hopkins considers the relationships between bimatrix games in which outcomes are ordinally ranked, but there may be ties between outcomes. Robinson and Goforth developed a group-theoretic classification for the no-tie case; Hopkins revisits this work from a graph-theoretic perspective and shows that the edges of an associated graph yield information for when there is a single tie. He expands the Robinson and Goforth system using a collection of simplices that includes all 2×2 ordinal rank games with ties.

Although the decomposition of Jessie and Saari in *Cooperation in n-Player Repeated Games* may be viewed as a type of classification, it is more of a tool to analyze behavior in games. For two-strategy, n-player games, they decompose the games' n-matrix payoffs into behavioral and strategic components. For a specific game in a repeated setting, the strategic component captures the information necessary to determine Nash equilibrium behavior, while the behavioral component describes a type of cooperative behavior. This approach lends itself to the description of all games with any specific type of behavior, and how this behavior changes as the components change.

In a bankruptcy problem, a set of individuals have claims that when summed exceed the amount of an estate. A bankruptcy rule determines how to share the estate among the claimants. Motivated by the relationship between the two-player Contested Garment rule and the n-player Talmud rule, Jones and Wilson (*The Dynamics of Consistent Bankruptcy Rules*) define a dynamic averaging process in which a k-player rule is used on all subsets of size k, the outcomes are averaged, and the processed is repeated. They show that when the k-player rule satisfies a well-studied notion of consistency, then the dynamic process converges to the n-player solution for any initial allocation.

In conclusion, as editors we want to thank all the authors for their interesting and strong papers. We enjoyed reading them, and hope that readers will experience that same pleasure in exploring the intersection between mathematics and the social sciences present in this volume.

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Decisions, Elections, and Games • Crisman et al., Editors

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