CONTEMPORARY MATHEMATICS

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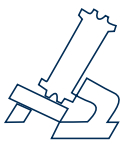
Israel Mathematical Conference Proceedings

Infinite Products of Operators and Their Applications

A Research Workshop of the Israel Science Foundation
May 21–24, 2012
Haifa, Israel

Simeon Reich
Alexander J. Zaslavski
Editors



American Mathematical Society
Providence, Rhode Island

Bar-Ilan University
Ramat-Gan, Israel

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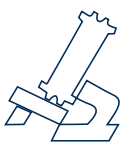

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Preface

A research workshop on Infinite Products of Operators and Their Applications took place at the Technion-Israel Institute of Technology, Haifa, Israel, on May 21–24, 2012. This workshop was organized and sponsored by the Center for Mathematical Sciences (CMS) at the Technion, with additional support from the Israel Science Foundation. The organizing committee was comprised of Simeon Reich (Technion) and Alexander J. Zaslavski (Technion). The workshop brought together a select group of over two dozen experts from four continents. The research workshop was devoted to various issues and open problems regarding infinite products of operators and their applications such as approximation theory, computed tomography, feasibility problems, generic convergence of algorithms, image recovery, optimization theory, population biology and the influence of computational errors.

This volume is the tangible record of this workshop. Most of the papers assembled here have been contributed by participants in the workshop. In some cases, they have chosen to submit manuscripts which depart from the texts of their lectures. Several invited speakers who were unable to attend the workshop also contributed papers to these proceedings. All submissions have been carefully refereed and revised whenever necessary.

The papers collected in this volume cover many different topics concerning infinite products of operators and their applications: projection methods for solving feasibility and best approximation problems, arbitrarily slow convergence of sequences of linear operators, monotone operators, proximal point algorithms for finding zeros of maximal monotone operators in the presence of computational errors, the Pascoletti-Serafini problem, remetrization for finite families of mappings, Poisson's equation for mean ergodic operators, vector-valued metrics in fixed point theory, contractivity of infinite products and mean convergence theorems for generalized nonspreading mappings.

The Editors

Conference Program

Monday, 21 May

09:00-09:50	Frank Deutsch *Arbitrarily slow convergence of sequences of linear operators*
09:55-10:45	Roger Nussbaum *The cone spectral radius, positive eigenvectors and Hausdorff dimension*
11:00-11:50	Roman Polyak *Nonlinear equilibrium for optimal resources allocation*
11:55-12:45	Jacek Jachymski *Some results on infinite products of operators involving the notion of joint spectral radius*
14:10-15:00	Ronald Bruck *Optimal constants for iterating monotone operators in Hilbert space using semi-definite programming*
15:30-16:20	Jon Borwein *Optimization techniques for inverse problems*

Tuesday, 22 May

09:00-09:50	Heinz Bauschke *Fixed point theory and monotone operator theory*
09:55-10:45	Amir Beck *Sparsity constrained nonlinear optimization: optimality conditions and algorithms*
11:00-11:50	Tadeusz Kuczumow *Theorems of Denjoy-Wolff type for a family of k_D-nonexpansive retracts*
11:55-12:45	Adi Ben-Israel *Product integrals of matrices*
14:00-14:50	Andrzej Cegielski *Methods for the variational inequality problem over the intersection of fixed point sets of quasi-nonexpansive operators*
14:55-15:45	Yair Censor *Using feasibility-seeking methods for superiorization*
16:00- 16:50	Adrian Petrusel *Vector-valued metrics in fixed point theory*
16:55-17:45	Shawn Wang *Restricted normal cones and the method of alternating projections*
17:50- 18:40	Adam Paszkiewicz *On the Amemiya-Ando conjecture*

Thursday, 24 May

09:00-09:50	Jon Borwein *Recent progress in maximal monotone operator theory*
09:55-10:45	Fran Aragon *Global convergence of a non-convex Douglas-Rachford iteration*
11:00-11:50	Shoham Sabach *Right Bregman nonexpansive operators in Banach spaces*
11:55-12:45	Michael Lin *On convergence of power series of contractions*
14:00-14:50	Evgeniy Pustylnik *Inclination of subspaces*
14:55-15:45	Sedi Bartz *Families of abstract convex antiderivatives*
16:00-16:50	Aviv Gibali *The split variational inequality problem*
16:55-17:45	Alexander J. Zaslavski *A proximal point method and a projected subgradient method in the presence of computational errors*

List of Participants

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