

CONTEMPORARY MATHEMATICS

643

Stacks and Categories in Geometry, Topology, and Algebra

CATS4 Conference

Higher Categorical Structures and their Interactions with
Algebraic Geometry, Algebraic Topology and Algebra

July 2–7, 2012

CIRM, Luminy, France

Tony Pantev
Carlos Simpson
Bertrand Toën
Michel Vaquié
Gabriele Vezzosi
Editors



American Mathematical Society

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Preface

The series of international conferences CATS has been running in the past several years and has brought together, in the convivial Mediterranean atmosphere, top level researchers from around the world interested in higher category theory and its applications to classical mathematical domains. It has been focused, from the very beginning, on the interactions of category and stacks theory with the other domains of the mathematical spectrum, from algebraic geometry, to topology, passing by number theory, algebra, representation theory, as well as mathematical physics. The topical diversity of the CATS meetings has attracted participants from very different mathematical fields making the scientific atmosphere of this series of conferences a rather unique one. Over the years, the conferences have hosted many participants of international recognition (R. Bezrukavnikov, K. Costello, D. Gaiotto, M. Hopkins, M. Kontsevich, J. Lurie, J. Morava, P. Schapira, B. Tsygan, M. Van den Bergh to mention a few), together with promising younger researchers, students and postdocs, in a friendly and relaxed atmosphere.

The first two conferences were held at the Université de Nice-Sophia Antipolis, during Fall 2001 and Spring 2003. For the third edition in Fall 2008, the conference moved to the Centro di Ricerca Matematica Ennio De Giorgi in Pisa, Italy. The fourth and largest of the CATS conferences took place in CIRM, Luminy, in the Summer of 2012.

The present volume comprises contributions of the participants to CATS4. Collectively the articles in the volume represent the state of the art of applications of categories and stacks to geometry, topology, algebra, representation theory and mathematical physics. The research presented here is on the forefront of the subject. Techniques such as localization, model categories, simplicial objects, sheaves of categories, mapping stacks, dg structures, hereditary categories, and derived stacks, are applied to give new insights on cluster algebra, Lagrangians, trace theories, loop spaces, structured surfaces, stability, ind-coherent complexes and 1-affineness showing up in geometric Langlands, branching out to many related topics along the way.

The paper of D. Calaque presents an application of the techniques of derived algebraic geometry to symplectic aspects of algebraic geometry and ultimately to the constructions and study of fully extended topological quantum field theories, and provides a link between homotopical algebra, derived algebraic geometry, low dimensional topology and mathematical physics. In the manuscript of J. Bergner and M. Robertson, higher categorical techniques, and more particularly the notion of $(1, \infty)$ -categories, are applied to produce a novel understanding of cluster algebras from the perspective of an algebraic topologist. An exciting relation between

higher Segal spaces, a particular kind of higher categorical structure, crossed complexes, and the topology of the moduli spaces of structured Riemann surfaces is presented in the contribution of T. Dyckerhoff and M. Kapranov. The paper of G. Vezzosi presents a homotopical approach to the notion of Lie algebroid, as well as its interpretations in the setting of derived algebraic geometry. Multiple intersections of Lagrangian submanifolds are studied, from the point of view of derived algebraic geometry, in the paper of O. Ben-Bassat, which explains how Lagrangian correspondences can be composed in the derived setting. In his paper, D. Gaitsgory presents the notion of 1-affineness, a categorical analog of the notion of being affine for an algebraic variety, and proves that an important class of derived algebraic stacks, natural for the geometric Langlands program, are indeed 1-affine. The manuscript of D. Kaledin contains a general study of traces in the higher and infinity categorical setting, with applications to the localization property of Hochschild homology. In their contribution, G. Dimitrov and L. Katzarkov study non semi-stable objects in derived categories, with applications to the representation theory of quivers. Finally, the paper of A. Preygel presents an approach to algebraic de Rham theory and algebraic D -modules based on the notion of Ind-coherent sheaves and derived loop spaces.

It is a pleasure for us to first thank all the participants in the CATS4 conference, for creating the nice productive atmosphere which, both from a scientific and social point of view, contributed to the great success of the event.

We would particularly like to thank the CIRM, Luminy, for accepting to host the conference, and for financial support through the housings of many of the participants. We also thank the mathematics department I3M (Université de Montpellier 2) and JAD (Université de Nice-Sophia Antipolis) for their financial supports. An important part of the funding for this event came from the ANR project "HODAG" (ANR-09-Blan-0151) and the NSF Research Training Group Grant DMS-0636606, and we warmly thank the ANR and NSF for their support.

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