

CONTEMPORARY MATHEMATICS

654

Centre de Recherches Mathématiques Proceedings

Rational Points, Rational Curves, and Entire Holomorphic Curves on Projective Varieties

CRM Short Thematic Program
Rational Points, Rational Curves,
and Entire Holomorphic Curves
and Algebraic Varieties

June 3–28, 2013

Centre de Recherches Mathématiques,
Université de Montréal, Québec, Canada

Carlo Gasbarri
Steven Lu
Mike Roth
Yuri Tschinkel
Editors



American Mathematical Society
Providence, Rhode Island

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Preface

Diophantine geometry and the study of rational points on algebraic varieties have greatly influenced and continue to revolutionize modern algebraic geometry. The general philosophy is that “the geometry determines arithmetic behaviour”.

It is conjectured that there are many rational points (at least after a finite extension of the base field) on “special” varieties, a class of varieties introduced by Campana which includes rationally connected and Calabi–Yau varieties. Conversely, the conjecture of Bombieri–Lang predicts that varieties of general type should have “few” rational points. These conjectures and philosophy are the subject of intense activity.

One of the fascinating aspects of these questions is their relations with complex analytic geometry. After Lang and Vojta, we expect that arithmetic properties of an algebraic variety correspond via value distribution theory to complex hyperbolic properties through a dictionary which translates properties of rational points into properties of holomorphic curves. For instance, for special varieties (in the sense of Campana) it is conjectured that the Kobayashi pseudo-distance is trivial, and that such varieties have many holomorphic curves, and in the simply connected case, many rational curves. Much work has been done to establish the expected properties which are the complex geometric counterpart to the above mentioned conjectured results in arithmetic.

In June 2013 a thematic month around these topics was organized at the CRM in Montreal supported in part by an ANR project grant. It was also generously supported by the NSF and locally supported by the CRM and CIRGET. Specialists from around the globe introduced the latest advances on the subject and specialized mini-courses were given geared to young researchers.

In this proceedings volume we gather the lecture notes of some of the mini-courses of the thematic month and contributed papers by key specialists in these areas.

Carlo Gasbarri
Steven Lu
Mike Roth
Yuri Tschinkel

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The Centre de Recherches Mathématiques (CRM) was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM receives funding from the Natural Sciences and Engineering Research Council (Canada), the FRQNT (Québec), the NSF (USA), and its partner universities (Université de Montréal, McGill, UQAM, Concordia, Université Laval, Université de Sherbrooke and University of Ottawa).

This volume contains papers from the Short Thematic Program on Rational Points, Rational Curves, and Entire Holomorphic Curves and Algebraic Varieties, held from June 3–28, 2013, at the Centre de Recherches Mathématiques, Université de Montréal, Québec, Canada.

The program was dedicated to the study of subtle interconnections between geometric and arithmetic properties of higher-dimensional algebraic varieties. The main areas of the program were, among others, proving density of rational points in Zariski or analytic topology on special varieties, understanding global geometric properties of rationally connected varieties, as well as connections between geometry and algebraic dynamics exploring new geometric techniques in Diophantine approximation.

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