# **CONTEMPORARY MATHEMATICS**

### 661

## Modern Trends in Constructive Function Theory

Constructive Functions 2014 Conference in Honor of Ed Saff's 70th Birthday May 26–30, 2014 Vanderbilt University, Nashville, Tennessee

> Douglas P. Hardin Doron S. Lubinsky Brian Z. Simanek Editors



**American Mathematical Society** 

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1098765432121 20 19 18 17 16 Dedicated to Ed Saff on the occasion of his 70th birthday

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#### Preface

This volume contains papers submitted by colleagues of Ed Saff and distinguished participants in the Constructive Functions 2014 Conference and School held from May 26 to May 30 at Vanderbilt University in Nashville, TN, USA. The conference and this volume are dedicated to Ed Saff on the occasion of his 70th birthday and aim to celebrate the influential role Ed has played in the fields of constructive function theory, approximation theory, potential theory, and applied mathematics. Many of these papers reflect the content of the talk one or more of the authors presented at the conference. The wide breadth of the subject areas discussed in this volume highlights the extent of Ed Saff's influence in mathematics over the last 40 years. It is a pleasure to dedicate this volume to him.

The Constructive Functions 2014 Conference and School featured over 140 participants from 30 countries. There were eleven plenary addresses complemented by more than 100 contributed talks in subjects ranging from random matrix theory and Riemann-Hilbert analysis to numerical analysis and approximation theory. Of particular note were the expository school lectures given by four prominent mathematicians and aimed at an audience of early career mathematicians. These 75-minute lectures were presented on Riemann-Hilbert analysis, numerical analysis and approximation, spectral theory on finite gap sets, and sphere packing and discrete minimal energy.

Without the tireless efforts of many individuals, the Constructive Functions 2014 Conference and School would not have been possible. While an accurate list would be too lengthy to include here, we must single out the contributions of the National Science Foundation (grant DMS-1363146), Springer Publishing, Pearson Publishing, the Society for Industrial and Applied Mathematics (SIAM), Loretta Saff, and the mathematics department staff at Vanderbilt University.

Special thanks are also due to Christine Thivierge and the technical staff of the American Mathematical Society for their efforts in creating this volume. Finally, we would like to thank the participants of the Constructive Functions 2014 Conference and School whose continuing efforts make this research field such an exciting frontier.

> Douglas P. Hardin Doron S. Lubinsky Brian Z. Simanek

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This volume contains the proceedings of the conference Constructive Functions 2014, held from May 26–30, 2014, at Vanderbilt University, Nashville, TN, in honor of Ed Saff's 70th birthday.

The papers in this volume contain results on polynomial approximation, rational approximation, Log-optimal configurations on the sphere, random continued fractions, ratio asymptotics for multiple orthogonal polynomials, the bivariate trigonometric moment problem, minimal Riesz energy, random polynomials, Padé and Hermite-Padé approximation, orthogonal expansions, hyperbolic differential equations, Bergman polynomials, the Meijer *G*-function, polynomial ensembles, and integer lattice points.



