

# CONTEMPORARY MATHEMATICS

672

## Topics in Functional Analysis and Algebra

USA-Uzbekistan Conference on  
Analysis and Mathematical Physics  
May 20–23, 2014  
California State University, Fullerton, CA

Bernard Russo  
Asuman Güven Aksoy  
Ravshan Ashurov  
Shavkat Ayupov  
Editors



American Mathematical Society

# Topics in Functional Analysis and Algebra



# CONTEMPORARY MATHEMATICS

---

672

## Topics in Functional Analysis and Algebra

USA-Uzbekistan Conference on  
Analysis and Mathematical Physics  
May 20–23, 2014  
California State University, Fullerton, CA

Bernard Russo  
Asuman Güven Aksoy  
Ravshan Ashurov  
Shavkat Ayupov  
Editors



---

American Mathematical Society  
Providence, Rhode Island

## EDITORIAL COMMITTEE

Dennis DeTurck, Managing Editor

Michael Loss      Kailash Misra      Catherine Yan

2010 *Mathematics Subject Classification*. Primary 17Axx, 17D92, 39Bxx, 42Bxx, 46A55, 46Lxx, 47A12, 47F05, 47Hxx.

---

### Library of Congress Cataloging-in-Publication Data

Names: Russo, Bernard, 1939- editor.

Title: Topics in functional analysis and algebra : first USA-Uzbekistan Conference on Analysis and Mathematical Physics, May 20-23, 2014, California State University, Fullerton, CA / Bernard Russo [and three others], editors.

Description: Providence, Rhode Island : American Mathematical Society, [2016] — Series: Contemporary mathematics ; volume 672 | Includes bibliographical references and index.

Identifiers: LCCN 2015047941 | ISBN 9781470419288 (alk. paper)

Subjects: LCSH: Topological algebras—Congresses. | Banach algebras—Congresses. | C\*-algebras—Congresses. | Functional analysis—Congresses. | AMS: Nonassociative rings and algebras – General nonassociative rings – General nonassociative rings. msc | Nonassociative rings and algebras – Other nonassociative rings and algebras – Genetic algebras. msc | Difference and functional equations – Functional equations and inequalities – Functional equations and inequalities. msc | Harmonic analysis on Euclidean spaces – Harmonic analysis in several variables – Harmonic analysis in several variables. msc | Functional analysis – Topological linear spaces and related structures – Convex sets in topological linear spaces; Choquet theory. msc | Functional analysis – Selfadjoint operator algebras ( $C^*$ -algebras, von Neumann ( $W^*$ -) algebras, etc.) – Selfadjoint operator algebras ( $C^*$ -algebras, von Neumann ( $W^*$ -) algebras, etc.). msc | Operator theory – General theory of linear operators – Numerical range, numerical radius. msc | Operator theory – Partial differential operators – Partial differential operators. msc | Operator theory – Nonlinear operators and their properties – Nonlinear operators and their properties. msc

Classification: LCC QA326 .T648 2016 | DDC 512/.55–dc23 LC record available at <http://lcn.loc.gov/2015047941>

Contemporary Mathematics ISSN: 0271-4132 (print); ISSN: 1098-3627 (online)

DOI: <http://dx.doi.org/10.1090/conm/672>

---

**Color graphic policy.** Any graphics created in color will be rendered in grayscale for the printed version unless color printing is authorized by the Publisher. In general, color graphics will appear in color in the online version.

**Copying and reprinting.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy select pages for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Permissions to reuse portions of AMS publication content are handled by Copyright Clearance Center's RightsLink<sup>®</sup> service. For more information, please visit: <http://www.ams.org/rightslink>.

Send requests for translation rights and licensed reprints to [reprint-permission@ams.org](mailto:reprint-permission@ams.org).

Excluded from these provisions is material for which the author holds copyright. In such cases, requests for permission to reuse or reprint material should be addressed directly to the author(s). Copyright ownership is indicated on the copyright page, or on the lower right-hand corner of the first page of each article within proceedings volumes.

© 2016 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights  
except those granted to the United States Government.  
Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines  
established to ensure permanence and durability.

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1      21 20 19 18 17 16

## Contents

Preface	vii
Tracial and Arens algebras associated with finite von Neumann algebras RUSTAMBAY ABDULLAEV	1
A dichotomy property for the graphs of monomials J. M. ALMIRA and Z. BOROS	9
Characterization conditions and the numerical index ASUMAN GÜVEN AKSOY and GRZEGORZ LEWICKI	17
On generalized localization of Fourier inversion for distributions RAVSHAN ASHUROV and ALMAZ BUTAEV	33
Derivations, local, and 2-local derivations on algebras of measurable operators SHAVKAT AYUPOV and KARIMBERGEN KUDAYBERGENOV	51
A survey on local and 2-local derivations on $C^*$ -algebras and von Neumann algebras SHAVKAT AYUPOV, KARIMBERGEN KUDAYBERGENOV, and ANTONIO M. PERALTA	73
Cohomology of Jordan triples via Lie algebras CHO-HO CHU and BERNARD RUSSO	127
Extreme boundary of the space of semi-additive functionals on the three-point set GAYRATBAY DJABBAROV	159
Algebras with genetic realization and corresponding evolutionary population dynamics NASIR GANIKHODJAEV	169
Quadratic homeomorphisms of the two-dimensional simplex and their trajectories R. N. GANIKHODZHAEV and A. T. PIRNAPASOV	179
On universal representations and universal enveloping locally $C^*$ -algebras for locally JB-algebras ALEXANDER A. KATZ and OLEG FRIEDMAN	185
Complex powers of the Schrödinger operator with singular potential ALIMDJAN KHALMUKHAMEDOV	205

Description of solvable Leibniz algebras with four-dimensional nilradical A. KH. KHUDOYBERDIYEV and Z. KH. SHERMATOVA	217
On classification problem of Loday algebras I. S. RAKHIMOV	225
On the classification of left-symmetric dialgebras IKROM M. RIKHSIBOEV	245
Invitation to research of new mathematics from biology: evolution algebras JIANJUN PAUL TIAN	257

## Preface

The USA-Uzbekistan Conference on Analysis and Mathematical Physics, focusing on contemporary issues in dynamical systems, mathematical physics, operator algebras and several complex variables, was hosted by California State University, Fullerton, on May 20–23, 2014. The main objective of the conference was to facilitate communication and collaboration between mathematicians from the USA and Uzbekistan. Zair Ibragimov, of CSUF, and Zafar Ibragimov of Urgench State University, were the key organizers of the conference, which featured invited speakers in mathematics, life sciences, physical sciences and engineering.

The present volume represents contributions from participants in the special session on Algebra and Functional Analysis, which was organized by the editors of this volume. Operator algebras served as a unifying theme for several papers in the volume.

Functional analysis is often used to study the analytic properties of functions on topological spaces and operators on Banach spaces. Reflecting the title of the conference, there are several contributions devoted to certain analytic topics, such as minimal projections with respect to numerical radius (Aksoy-Lewicki), and functional equations and discontinuous polynomials (Almira-Boros). Other contributions in functional analysis were concerned with Fourier inversion for distributions (Ashurov-Butaev), Schrödinger operators (Khalmukhamedov), convexity (Djabbarov) and dynamical systems (Ganikhodzhaev-Pirnapasov).

The theory of von Neumann algebras was initiated as a series of papers by Murray and von Neumann in the 1930s and 1940s. The study of  $C^*$ -algebras was begun in the work of Gelfand and Naimark in the 1940s. Since then, the subject of operator algebras has evolved into a huge mathematical enterprise interacting with almost every branch of mathematics and several areas of theoretical physics.

To be sure, the study of operator algebras depends on some basic as well as advanced algebraic topics, including those of the nonassociative variety. Accordingly, this volume also contains some contributions in pure and applied algebra, such as Leibniz algebras (Rakhimov, Khudoyberdiyev-Shermatova), genetic and evolution algebras (Ganikhodjaev, Tian), and low dimensional classification of dialgebras (Rikhsiboev), as well as some operator algebraic contributions which depend on Jordan structure (Katz-Friedman, Chu-Russo).

One of the main themes of the contributions on von Neumann algebras concerned derivations. Derivations appeared for the first time at a fairly early stage in the field of  $C^*$ -algebras, and their study continues to be one of the central branches in the field. The study of bounded derivations has led to a beautiful mathematical theory that provides the essential tools for the study of unbounded derivations,



which in turn is motivated by the problem of constructing the dynamics in statistical mechanics. In the words of a pioneer in the field:

“A veritable army of researchers took the theory of derivations of operator algebras to dizzying heights—producing a theory of cohomology of operator algebras as well as much information about automorphisms of operator algebras.” —*Richard Kadison (Which Singer is that? 2000)*

In the spirit of this quotation, and reflecting the fact that the structures of Jordan derivations and Lie derivations on von Neumann algebras are now well understood, a study of the higher dimensional nonassociative cohomology of a von Neumann algebra was initiated in the contribution of Chu-Russo.

The properties of derivations on algebras of unbounded operators are far from those of derivations on  $C^*$ -algebras and von Neumann algebras, which, as noted above, are well behaved and understood. Ayupov initiated the study of these problems in this context and his contribution to this volume, jointly with Kudaybergenov, surveys the latest results in this area on algebras of measurable operators. In another direction, the extensive survey on local and 2-local derivations (the latter are, *a priori*, not assumed to be linear or continuous) on  $C^*$ -algebras and  $JB^*$ -triples (Ayupov-Kudaybergenov-Peralta) summarizes and extends recent developments in this setting.

Bernard Russo (Irvine)  
Asuman Güven Aksoy (Claremont)  
Ravshan Ashurov (Tashkent)  
Shavkat Ayupov (Tashkent)

The USA-Uzbekistan Conference on Analysis and Mathematical Physics, focusing on contemporary issues in dynamical systems, mathematical physics, operator algebras, and several complex variables, was hosted by California State University, Fullerton, from May 20–23, 2014. The main objective of the conference was to facilitate scientific communication and collaboration between mathematicians from the USA and Uzbekistan.

This volume contains the proceedings of the Special Session on Algebra and Functional Analysis. The theory of operator algebras is the unified theme for many papers in this volume. Out of four extensive survey papers, two cover problems related to derivation of various algebras of functions. The other two surveys are on classification of Leibniz algebras and on evolution algebras. The sixteen research articles are devoted to certain analytic topics, such as minimal projections with respect to numerical radius, functional equations and discontinuous polynomials, Fourier inversion for distributions, Schrödinger operators, convexity and dynamical systems.

ISBN 978-1-4704-1928-8



9 781470 419288

CONM/672

AMS on the Web  
[www.ams.org](http://www.ams.org)