CONTEMPORARY MATHEMATICS

679

Recent Progress on Operator Theory and Approximation in Spaces of Analytic Functions

Conference on
Completeness Problems, Carleson Measures,
and Spaces of Analytic Functions
June 29–July 3, 2015
Institut Mittag-Leffler, Djursholm, Sweden

Catherine Bénéteau
Alberto A. Condori
Constanze Liaw
William T. Ross
Alan A. Sola
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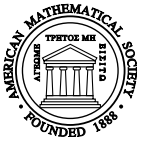

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Preface

This volume contains the proceedings of the conference *Completeness problems, Carleson measures, and spaces of analytic functions* that took place at the Institut Mittag-Leffler (IML), Sweden, from June 29 to July 3, 2015.

The conference brought together experienced researchers and promising young mathematicians from many countries to discuss recent progress made in function theory, model spaces, completeness problems, and Carleson measures. It consisted of roughly 14 hours of lectures, an open problems session, and discussion periods that promoted collaboration among the participants.

In these refereed proceedings, we have included contributed articles that have not appeared elsewhere, both on cutting-edge research questions as well as longer survey papers, and a report on the problem session that contains a collection of attractive open problems in complex and harmonic analysis.

We would like to take the opportunity to thank the IML staff and Director, Ari Laptev, for providing an outstanding venue that allowed the development of informal collaborations among the participants. In addition, we are grateful to the National Science Foundation for their support of the US participants, in particular graduate students, post-docs, and pre-tenure faculty. We would also like to thank all the authors who submitted articles to this collection. A special thanks goes to Alexandru Aleman, Emmanuel Fricain, Alexei Poltoratski, Rishika Rupam, and Kristian Seip, for their presentations at the problem session at the conference as well as their written contributions to the volume. We also thank the anonymous referees whose prompt and thorough reviews improved the quality of the proceedings. Finally, we thank the participants of the conference whose enthusiastic participation made the event a great success.

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Constanze Liaw
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